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ABSTRACT

Described are the implemented and attained mathematics curriculum of 18 countries that participated in the Second International Mathematics Study. Differences and similarities between countries, are illustrated through analysis of the data, and data are presented to indicate shortcomings in the content and outcomes of education within certain countries. The bulk of the document consists of two appendixes consisting respectively of 180 test items and a data matrix. (Author/PK)

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Second International Mathematics Study

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The Implemented and Attained Mathematics Curriculum
A Comparison of Eighteen Countries

Center for Education Statistics
Office of Educational
Research and Improvement
U.S. Department of Education

Contractor's Report

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Second International Mathematics Study

The Implemented and Attained Mathematics Curriculum:
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July 1986

ABSTRACT

In this paper a description is given of the implemented and attained mathematics curriculum of eighteen countries who participated in the Second International Mathematics Study. The aim of this paper is to illustrate the kind of differences and similarities between countries, which can be found by analyzing the collected data. Within clusters of countries with comparable implemented curricula reference data can be found from which shortcomings can be identified in the content and outcomes of education within certain countries.

INTRODUCTION

Many evaluation studies in different countries are directed at describing the educational situation in certain parts of the school system or at estimating the effect of certain educational measures for the improvement of the educational process and its outcomes. An understanding of the whole of various measures can be gained by performing periodical assessment studies which cover the total schoolsystem. One special type of assessment studies, performed periodically, are the international comparative studies of IEA (International Association for the Evaluation of Educational Achievement). These studies are focussed on certain schoolssubjects and enable an evaluation of education on the national level in comparison with other educational systems. International empirical studies are important for at least three different reasons:

1. It enables a description of differences and similarities between national educational systems and enables the identification of specific idiosyncrasies within particular countries.
2. Comparison of the results of a country with relevant others may result in the identification of weak areas for which measures to optimize education could be developed.
3. It contributes to the understanding of how education functions in a variety of different settings.

The Second International Mathematics Study (SIMS) was one of the IEA-studies in which 20 countries participated (from 1977 - 1984). In this report we will explore some possibilities for using the international data of the SIMS to identify shortcomings within national educational systems. Attention will be especially focussed on data regarding the implemented and attained mathematics curriculum.

IEA is an international organization with about 40 member countries. Since the early sixties IEA has been involved in multinational research projects. At first, attention was focussed on the study of the outcomes of education in several disciplines. In recent projects a wider range of educational research questions such as the causes of early school leaving on the influence of the classroom environment has been studied. Twelve countries took part in IEA's first project: the first mathematics project. The results of this study are reported internationally by Husen (1967).

In the period 1970-1975 the Six Subject Study was undertaken. In this study reading comprehension, science, civics, English (as a foreign language), French (as a foreign language) and literature was investigated. The results of this study are reported in 9

volumes of the International Studies in Evaluation. In order to make comparisons between countries which provide optimal information, one has to be careful in the choice of the research design and of the instruments. E.g. no differences in total-testscores sometimes mask really interesting differences at subtest or item level, so only looking at total test scores may produce anti-information. One may however add to this that even comparisons of student achievement on subtest and/or item level may be trivial if the implemented curriculum (which is the subject matter in which the students really were taught) is not taken into account. In this paper we will present possibilities of country comparisons based upon an analysis of the implemented and attained curriculum simultaneously. Our aim is primarily to develop a method for country comparisons which allows for maximal information as a basis for identifying the areas for which optimization measures could be taken. After a description of the background and the design of the study and the data which were analyzed we will first describe similarities and differences between countries on the attained and implemented curriculumlevel. Finally comparisons between countries will be made based upon analysis of data on the two levels simultaneously.

BACKGROUND OF THE SECOND MATHEMATICS STUDY

In the sixties important changes in the mathematics education took place all over the world. Changing opinions about the content and the didactics of school mathematics were the starting point of a profound revision of the mathematics curricula. In many countries these developments stabilized in the beginning of the seventies. The second part of this decade was therefore a good period for a state-of-the-art study of mathematics in the schools. The major aim of the project is to give a description of the relationship which exist between (a) the mathematics program (what is the content and context of mathematics teaching?), (b) the affective and cognitive results of the students (what is the output of mathematics teaching?) and (c) the teaching-learning process (in what way is the output achieved?). We can study the mathematics curriculum on three different levels. On the first level we have the intended curriculum, as specified in the official documents of a country. The second level is the curriculum as implemented within the schools and the classrooms. In the actual mathematics lessons the intended curriculum is given its concrete form. Here the time to be spent on the parts of the curriculum, the didactics and the methods are determined. Finally, we have the attained curriculum: the (affective and cognitive) objectives the students have attained. In the study the content of each of these levels is described and the relationships between them are investigated. Each curriculum level is a special object of study in certain parts of the SIMS (see figure 1). In this figure is also indicated on which level data were collected.

Component of Study	Object of Study	Data from
I Curriculum analyses	Intended Curriculum	Countries (education systems)
II Classroom processes	Implemented Curriculum	Schools and Classes
III Outcomes	Attained Curriculum	Students

Figure 1: Schematic overview of the study.

In the phase curriculum analysis, attention has been paid to the content (i.e. the topics in school mathematics) and the context (e.g. school systems, examination system) of the intended mathematics curriculum. In this paper we will not deal with these analyses; see Steiner (1980) for the first results. The study of the teaching-learning processes within the classroom is (amongst others) aimed at the description of the implemented curriculum, the methods used and the didactics applied in these methods. In the third part of the study the cognitive and the affective results of the students are assessed in relation to the intended and the implemented curriculum and several other variables (e.g. hours spent on home work and gender).

SUMMARY DESIGN AND INSTRUMENTS

In the next sections only those characteristics of the design of the study are mentioned which are necessary for a good understanding of the results presented later.

The Design of the Study

A total of 20 countries participated in the SIMS. The design of the study was a result of discussions between the participating countries. Each country could take part according to the complete international design or only in parts of the study. In this paper we will restrict ourselves to one of the two internationally proposed populations. The international definition of this population (population A) is:

all students in the grade level where the majority has attained the age of 13.00 - 13.11 by the middle of the school year. In most countries this population is the second year of secondary education (US-grade level 8).

In each country a representative sample of students from this population was drawn.

Instruments

The following test and questionnaires were used:

1. Cognitive tests
2. Student background questionnaires
3. Teacher questionnaire "Opportunity to Learn"
4. Teacher background questionnaires
5. School questionnaire

For this paper especially the instruments 1. and 3. are of importance. The cognitive tests are important instruments to measure the attained curriculum. They consist of five-choice items from five content areas (Arithmetic, Algebra, Geometry, Statistics, Measurement). Each student answered a part of the items, by taking a test of app. 40 items, which was the same for all students (core test), and one of the four tests (of app. 34 items), each of which was designed for a quarter of the students (rotated forms). The "Opportunity to Learn" questionnaire is one of the instruments to measure the implemented curriculum. In this questionnaire several questions are posed to investigate whether the subject matter, represented by the respective items, was taught to the students or not. In other words: did the students have an opportunity to learn (OTL) the subject matter represented by that item? In the international wording of the question for each item teachers had to indicate in which of the following periods the subject matter concerned was or should be taught:

1. Before this year.
2. This year (before the day of testing).
3. Never or after this year.

For most countries "this year" is the second year of secondary education (the 8th of compulsory education). There are exceptions, because in some countries an age based sample instead of a grade based sample was used.

To eliminate from this rating the hidden estimation of the difficulty of the item for a particular class, the teacher was also asked to estimate (per item) the percentage of students in his/her class who should be able to answer the item correctly without guessing.

THE DATA

The data which are reported in this paper stem from the following 18 countries:

1 Belgium-Flemish	10 Israel
2 Belgium-French	11 Japan
3 Canada-BC	12 Luxembourg
4 Canada-Ont	13 Netherlands
5 England	14 New Zealand
6 Finland	15 Scotland
7 France	16 Sweden
8 Hongkong	17 Thailand
9 Hungary	18 USA

The content of the cognitive tests was not the same for all countries, because some countries took part in the so called longitudinal component of the study in which the same students were tested on different occasions, while other countries only participated in the cross-sectional component of the study, in which students were tested just once. The item sets for both components were not completely overlapping. The sequence of items in both study components was different. We will restrict our analyses to the 157 items which were common in both parts of the study. These items are listed in appendix I. After the proper data modifications for each item a weighted percentage correct and percentage OTL (which was calculated by counting the answers in the categories "this year" and "before this year") was computed. In appendix II these percentages are listed for each item and country. The weights consisted of the stratum weights which were available on the international datafiles. For three countries (Belgium-French, Hongkong, Scotland) OTL-ratings were not available. Table 1 contains some overall statistics for each country. The first six columns give test results, the latter six OTL-percentages. The first of these sets of columns contains total test results, while the others represent subtests, respectively for the subjects arithmetic (ARIT), algebra (ALGB), geometry (GEOM), statistics (STAT) and measurement (MEAS). In table 1 interesting phenomena can be noted. First of all the table shows that Japanese students have the highest achievement scores on the total test and the five different subtests. For other countries the achievement-level is not so consistent for all subtests. For instance: the mean score of the USA is very low, but this is mainly caused by low scores in Algebra, Geometry and Measurement (which also have relatively low OTL's). The same holds for Luxembourg and Sweden. In France and Israel the subscores on Geometry are very low. It is noteworthy to see that at the same time the OTL for this subdomain is also very low in these countries. In some countries (e.g. Thailand and Hungary) the OTL-scores on certain subtests are very high while the achievement of students is on a level of countries who have a much lower OTL. This observation may lead these countries to a further analysis of their data, searching for possible causes which may lead to measures for improvement. When OTL is a good predictor of achievementlevel we might expect that the relative position of countries in the OTL-

Table 1: mean percentages correct scores (TEST) and mean percentages Opportunity to Learn (OTL) on subtests and the total test.

TEST

COUNTRY	TOTAL	ARIT	ALGB	GEOM	STAT	MEAS
1 Belgium-Flemish	52	57	51	42	58	58
2 Belgium-French	52	58	51	44	53	57
3 Canada-BC	51	57	48	42	60	52
4 Canada-Ont	49	54	42	42	56	51
5 England	47	48	39	44	60	48
6 Finland	50	49	46	45	61	54
7 France	53	58	55	38	57	60
8 Hongkong	49	55	43	43	56	53
9 Hungary	57	57	51	54	61	62
10 Israel	45	51	46	36	53	47
11 Japan	62	60	60	57	71	69
12 Luxembourg	39	48	34	26	39	52
13 The Netherlands	58	60	52	53	67	63
14 New Zealand	46	46	40	45	58	46
15 Scotland	49	51	43	46	60	49
16 Sweden	44	43	34	40	60	52
17 Thailand	43	44	38	40	46	49
18 USA	45	51	43	38	58	41

OTL

COUNTRY	TOTAL	ARIT	ALGB	GEOM	STAT	MEAS
1 Belgium-Flemish	62	78	75	31	39	84
2 Belgium-French	-					
3 Canada-BC	65	78	79	46	45	72
4 Canada-Ont	72	88	71	51	61	85
5 England	71	81	66	56	71	82
6 Finland	59	73	67	36	50	68
7 France	74	89	89	44	52	95
8 Hongkong	-					
9 Hungary	88	89	90	85	84	95
10 Israel	57	66	77	38	45	57
11 Japan	75	83	82	51	74	93
12 Luxembourg	55	75	48	33	31	78
13 The Netherlands	68	78	70	64	31	79
14 New Zealand	67	70	66	63	62	74
15 Scotland	-					
16 Sweden	50	63	43	34	44	64
17 Thailand	73	84	81	56	52	84
18 USA	68	84	67	44	70	74

ranking is approximately the same as in the ranking by mean percentages correct. Figure 2 shows this relation for the 15 countries on the total test for which both mean achievement scores and OTL's were available.

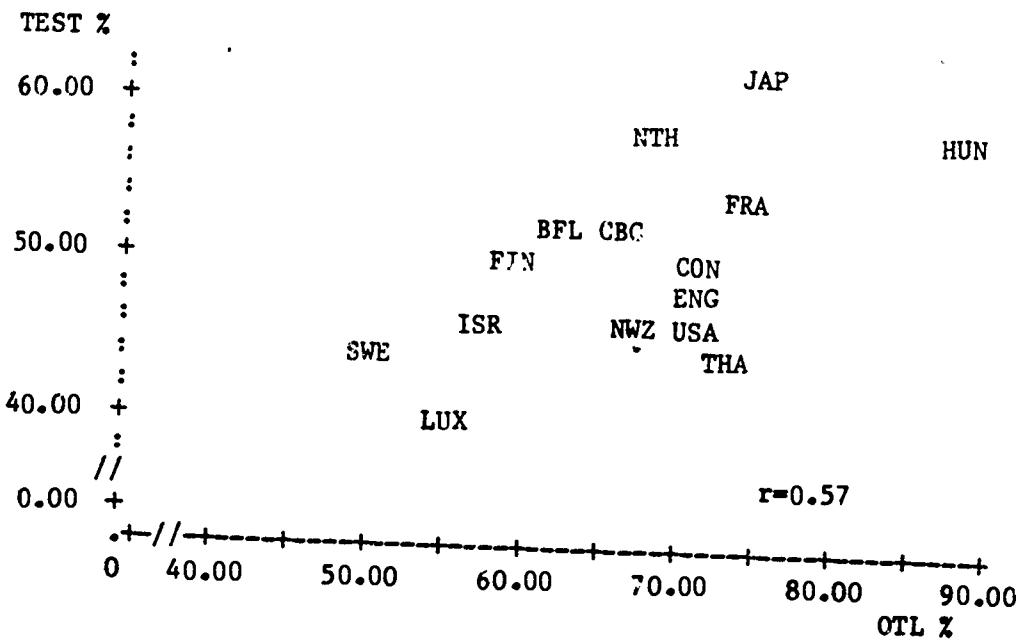


Figure 2: scattergram of testscores (TEST) and percentages opportunity to learn (OTL) for 15 countries

Figure 2 clearly shows that there is a relation: countries with a high OTL in comparison with countries with a low OTL on the average have students with higher achievement scores. The figure however also shows that countries with approximately comparable OTL's can have very different achievement scores. This means that besides OTL also other factors are influencing the outcomes of education. We will explore the data further by looking at similarities and differences between countries at a more detailed level.

SIMILARITIES AND DIFFERENCES BETWEEN COUNTRIES

In studying the similarities and differences between countries, we will adopt an approach whereby we will work with data on the item level. Although this approach has the disadvantage that we have to be very cautious not to capitalize on idiosyncrasies of single items, the advantage is that it is focussed on the most concrete level of mathematics content which in this study is possible. This means that we circumvent the disadvantage of working with predefined categorizations which are merely legitimated in terms of the structure of the subjectmatter, but not in terms of empirical observable phenomena. In the following we will try to explore whether an item level approach yields interpretable results. From the patterns of percentages correct and of OTL we can learn what countries have in common and to what degree differences exist. Using the set of 157 items the correlation between countries percentages correct responses on the items were calculated.

The same has been done for the percentages OTL. The first correlation shows the degree to which the items which were relatively easy (or difficult) in one country have the same relative easiness (or difficulty) in other countries. The same holds for OTL-correlations. Table 2 contains all the different intercorrelations which could be obtained in this way: the lower triangle contains the correlations of percentages correct across the items while the upper triangle represents correlations of OTL-percentages.

Table 2: intercorrelations (x100) between countries of percentages correct (under triangle) and percentages OTL (upper triangle)

	BFL	CBC	CON	ENG	FIN	FRA	HUN	ISR	JAP	LUX	NTH	NWZ	SWE	THA	USA
BFL	-	53	64	55	68	80	34	59	59	73	50	39	52	64	62
CBC	78	-	74	63	63	54	37	75	46	60	62	49	58	66	73
CON	79	93	-	83	73	61	51	71	58	80	68	63	73	80	92
ENG	71	85	84	-	73	52	54	64	65	74	68	76	76	64	81
FIN	77	78	79	81	-	72	50	70	67	64	66	70	75	62	73
FRA	85	78	76	67	75	-	46	63	69	68	51	41	51	63	61
HUN	71	75	77	78	83	70	-	51	70	39	50	53	50	59	44
ISR	79	80	77	76	79	77	80	-	66	61	62	48	56	73	72
JAP	69	67	66	69	75	68	76	72	-	61	48	48	52	60	56
LUX	78	70	69	63	68	80	70	81	66	-	68	49	67	67	74
NTH	75	82	80	83	85	72	81	81	70	72	-	72	64	64	53
NWZ	70	87	86	92	78	64	72	70	67	56	79	-	66	48	56
SWE	69	71	73	82	84	63	75	70	64	63	77	75	-	65	73
THA	73	79	80	77	73	69	77	74	70	68	67	76	71	-	75
USA	76	92	94	84	77	75	71	77	64	54	77	85	69	73	-

Table 2 shows that the intercorrelations of the percentages correct generally are high (in 99 of the 105 cases the test correlation is higher than the OTL correlation) and that the OTL-correlations show more variability (shown e.g. by the range), illustrating that the OTL profiles of some countries resemble each other more than the profiles of other countries.

For instance, the implemented curriculum (as measured by the IFL-ratings) in Belgium-F1 corresponds most with that of France and Luxembourg and less with that of New Zealand and Hungary. The implemented curriculum of New Zealand on its turn is most closely associated with that of England and The Netherlands. The USA curriculum looks most alike that of Canada-Ontario and England. A factoranalysis reveals some groupings of countries (see the plot in figure 3). The biggest contrast is formed by the groups France, Belgium-F1 and Japan versus Canada-Ont, Canada-BC and the USA.

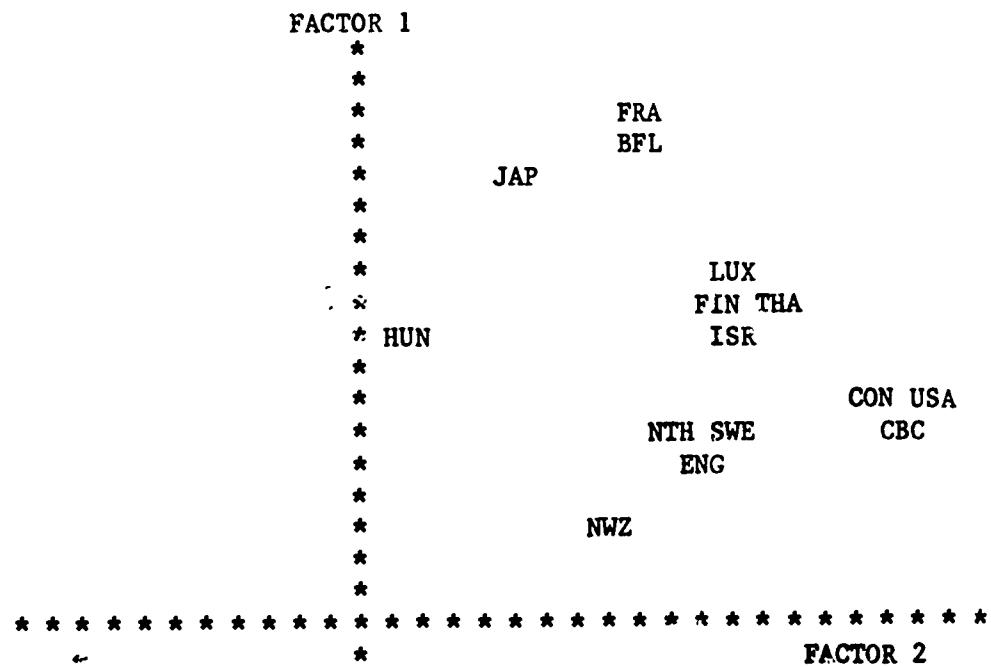


Figure 3: plot of first two factors after principal component analysis and varimax rotation on OTL-percentages.

An inspection of the differences between these two groups at the itemlevel reveals that in the Belgium/Japanese/France group there is a high emphasis on arithmetic in the form of wordproblems and a low emphasis on the theorem of Pythagoras and square roots, while in the Canadian/USA group there is a high emphasis on arithmetic in the form of calculations and a low emphasis on vector geometry (represented what in SIMS often are called "The French items"). The analysis (and therefore its graphical representation)is meaningful, because it confirms some broadly expected curricular distinctions.

It is however important to remember that correlations and factoranalysis are only sensitive for the relative ordering, so what the correlations in table 2 and figure 3 are showing is the degree of correspondence between countries as far as the relative emphasis on teaching the items is concerned.

However, countries which look alike very much in this way, may differ a lot when one looks at the absolute differences in OTL-ratings. In order to show this, we calculated for each item and each pair of countries the absolute difference of OTL-percentages and summed these over all items. In table 3 these sums(divided by 100)for each pair of countries are displayed. The table shows that still much of what was visible in the correlation table, is also present in this table of absolute differences, but it gives more information how close or apart countries are.

Table 3: sum of absolute differences of OTL-percentages between countries (divided by 100).

Table 3 shows for instance that the USA and Canadian-Ont implemented curriculum really resemble each other very much: the sum of absolute differences is 1300 calculated over 157 items, which means an average difference of 4.08 in OTL percentages per item over these two countries. It illustrates however also that other countries which had high intercorrelations of OTL-values and which are in the same group as a result of the principal component analysis may still have a lot of differences. An example is BFL and LUX, for which the sum of absolute differences is 2800, which is more than two times as high as in the preceding example. How consistent and meaningful the differences between countries are, can be discovered by inspection of the item content of the items for which the difference in OTL is relatively great, e.g. greater than 30%. The result of one of the possible 196 comparisons (in this case

USA and Japan) is displayed in table 4, which shows that in the Japanese curriculum goniometry, coordinates, calculation of surface area and content and formulaes are more emphasized than in the USA, while in the USA arithmetic, square roots and the properties of geometric figures are more emphasized. What is

Table 4: item contents and OTL in Japan and the USA for items which differ more than 30% in OTL

ITEMCONTENT	DIFFERENCE	OTL	
		JAP	USA
3 IF $5X+4=4X-31$ THEN X EQUALS	41	97	56
7 FLAT CARDBOARD CUBE	65	91	26
11 MIDPOINT OF NUMBERLINE	32	87	55
13 CIRCUMFERENCE OF CIRCLE	69	92	23
27 GIVEN X KG OF TEA. SELL 15 KG	52	96	44
30 DERIVE RELATION FROM TABLE	35	78	43
42 REFLECTION OF LINE	31	58	27
54 PARALLEL LINES	59	66	7
55 CORNER FROM WOODEN CUBE.VIEW ABOVE	49	68	19
56 INFER VALUES OF P AND Q IN TABLE	32	95	63
72 SAVE 3 OR 5 \$. HOW MANY MONTHS 10 MORE	32	94	62
74 WHICH POINT JOIN TO $(-3,4)$ NOT CUT X/Y AXIS	45	82	37
79 ANGLE OF CIRCLE GRAPH	65	95	30
81 ANGLE OF BCD	35	69	34
93 AREA OF FIGURE	37	95	58
122 DERIVE FORMULA FROM GIVEN DATA	37	96	59
123 DERIVE FORMULA FROM GIVEN DATA	35	96	61
127 HOW MANY BLOCKS IN BOX OF GIVEN SIZE	31	97	66
129 RING TOGETHER BELLS WITH DIFFER. INTERVALS	50	96	46
130 SURFACE AREA OF RECTANGULAR BOX	35	97	62
141 AREA OF GIVEN FIGURE	33	98	65
145 $A/15 - B/5$ IS EQUAL TO	48	87	39
152 ESTIMATION OF AREA IN SHADED REGION	34	81	47
164 SIZE OF ANGLE BCD	38	82	44
167 RESULT AFTER ROTATION OF FIGURE	31	53	22
172 DERIVE FORMULA FROM GIVEN DATA	59	96	37
40 SIMILAR TRIANGLES. HOW LONG IS SU?	- 44	4	48
70 SQUARE ROOT OF 12×75 .	- 57	1	58
73 0.00046 IS EQUAL TO	- 56	15	71
85 $3/5 / 2/7$ IS EQUAL TO	- 64	36	100
97 DERIVE N FROM EXPONENTIAL EQUATION	- 62	6	68
100 THEOREM OF PYTHAGORAS	- 54	1	55
108 SQUARE ROOT OF 75	- 61	0	61
111 THEOREM OF PYTHAGORAS	- 54	2	56
116 PROBABILITY SELECTING RED BUTTON FROM JAR	- 33	6	39
119 DEFINITION SIMILAR TRIANGLES	- 44	3	47
133 DEFINITION PARALLELOGRAM	- 54	20	74
143 SINCE $4X9=36$, SQUARE ROOT 36 IS EQUAL	- 60	0	60
171 $X/2 < 7$ IS EQUIVALENT TO	- 34	20	54

moreover noteworthy in table 4 is the consistency of ratings for items which have a comparable content, e.g. item 100 and 111 (Pythagoras) or items 122, 123 and 172 (derive formula from given data). The same kind of comparisons can be made for other combinations of countries, which may result in a description of how countries differ in emphasizing certain subjectmatter in their implemented curriculum. What the data show is the diversity which exists in implemented curricula of different countries. When comparisons between countries are made with respect to achievement data these differences in implemented curricula should be taken into account.

THE IDENTIFICATION OF WEAK AREAS

As one of the goals of the SIMT is to contribute to the improvement of education, one may try to find in the data the areas in which student performance might be improved. As there are no absolute standards to make these kind of judgements a relative approach has to be sought. In the preceding sections we showed that comparisons between countries have to take account of the diversity of OTL. In this paragraph we want to explore what countries may learn from the achievement results of other countries by looking simultaneously at achievement and OTL data. As we are in this paper exploring a possible method we choose for an approach for comparing countries using rather conservative criteria. This is done by considering for each country only those items which have a large OTL (more then 80%). Furthermore we will only consider those items for which there is a large difference of p-value of a country with the country of reference (we choose a difference larger then -20%). The number of items which suffice these conditions for each pair of comparisons is shown in table 5. From the table we may see that e.g. there is one item with high OTL in Canada-BC and in Belgium-F1. on which Canada-BC students perform less (according to this criterion) than the students from Belgium-F1. and that there are four items on which the Canada-Ont students perform less then the Belgium students, etc.

Tabel 5: number of items which suffice the condition that
 OTL-country and OTL-reference country $>80\%$ and
 p-value-country minus p-value-reference-country $< -20\%$.

REFERENCE COUNTRIES

	BFL	CBC	CON	ENG	FIN	FRA	HUN	ISR	JAP	LUX	NTH	NWZ	SWE	THA	USA
BFL	0	0	0	0	2	4	0	10	0	1	0	0	0	0	0
CBC	1		1	0	1	6	2	2	6	2	3	0	0	0	1
CON	4	0		0	1	8	7	1	17	2	10	0	1	0	0
ENG	10	7	4		6	18	15	3	22	4	18	0	2	1	1
FIN	5	2	1	0		6	1	0	8	0	3	0	0	0	0
FRA	2	3	2	0	2		11	1	11	1	8	2	1	1	1
HUN	2	1	0	0	0	5		0	12	0	4	1	0	1	0
ISR	1	1	0	0	0	4	1		7	0	1	0	0	0	0
JAP	2	0	1	0	0	1	1	0		1	1	0	0	1	0
LUX	4	2	2	0	0	1	6	0	11		2	0	1	0	0
NTH	0	0	1	0	0	5	1	0	5	0		0	0	0	1
NWZ	10	5	5	0	5	18	13	3	20	7	16		1	0	1
SWE	4	5	3	0	2	6	4	1	8	1	5	1		1	2
THA	16	7	8	2	9	25	22	4	37	7	17	1	2		6
USA	7	2	1	1	3	14	9	2	21	1	10	1	2	2	
	BFL	CBC	CON	ENG	FIN	FRA	HUN	ISR	JAP	LUX	NTH	NWZ	SWE	THA	USA
OTL>80	72	57	69	65	44	92	134	20	101	41	63	53	33	84	62

Table 5 shows that the number of items for several comparisons may differ considerably. It is however important to stress that in some cases this is due to the absence of items which fulfill the OTL>80% condition, so a country with a very heterogeneous curriculum and consequently no items with OTL>80% is hardly represented with items in this analysis and consequently doesn't have many countries to compare with. Therefore the bottom-row of table 5 shows the number of items which fulfill the condition OTL>80% within each country. The number of these items are relatively low for Israel, Sweden, Luxembourg and Finland (which is consistent with the overall statistics in table 1).

So the less the number of items with OTL>80% , the less the chance that there will be items which fulfill all the conditions for this analysis. This disadvantage might be circumvented by calculating within each country p-values only for those students which had an OTL for that item. However this calculation cannot be done straightforward as a number of other variables have to be controlled simultaneously in order to prevent unfair comparisons.

In table 6 for each country a short description is given of the information which is available through table 5 and the content of the items. We repeat that our analysis is a conservative one and only reveals the areas in which relatively poor achievement is occurring.

Table 6: description of weak areas/items per country

COUNTRY	WEAK AREAS/ITEMS
Belgium-Flemish	the calculation of areas of plane figures and surface areas and volumes of solids.
Canadian-BC	the calculation of areas and arithmetic based on word problems.
England	fractions, exponents, multiplication with decimal numbers, reading of scales, arithmetic based on word problems, calculation of areas and volumes, simplification of algebraic expressions
Finland	fractions and the subtraction of negative numbers.
France	calculation and estimation of areas, percentages and fractions
Hungary	fractions and the calculation of areas
Israel	13: if $P=LW$, $P=12$, $L=3$ then $W=?$ 166: if $x=-3$ then $-3y=?$ 173: $7 \times (3 + 9)$ is equivalent to
Japan	1: 2 meter + 3 millimeter equals 11: find the midpoint of a line segment 33: given 300 girls and 800 students, what is ratio boys/girls 125: capacity of cubic container of $10 \times 10 \times 10$ cm in liters
Luxembourg	calculation of areas of figures, volume of solids and calculations of fractions.
Netherlands	calculation of fractions and exponents, with the subtraction of negative numbers.
New Zealand	fractions, negative numbers, decimal numbers, arithmetic based on word problems and numberlines.
Sweden	fractions, subtraction of large numbers and with numberlines
Thailand	a large number of different items.
United States	arithmetic based on word problems, the calculation or estimation of areas, the multiplication of negative numbers, fractions and percentages and with numberlines.

The description above indicates in which areas probably improvement measures could be taken. It is however important to realize that the items which we mentioned are probably only a subset of items which point to areas in which underachievement occurs, because the selection of items is based upon estimates of OTL at the national level. Therefore it would be advisable, before considering such measures to carry out more detailed analyses and look at the performance of certain groups of students within a country (e.g. 10% best, 10% worse or students from certain types of schools) in comparison with other countries. This could give a better understanding of the question where underachievement is located. After that the question should be raised why student performance on a particular subset of items in a certain country is relatively low. These analyses are however beyond the scope of this article.

CONCLUSIONS

In this paper we explored a method of country comparisons which takes account of the fact that the mathematics curriculum differs between countries. Our calculations show that it is important to take into account for any comparison of cognitive measures the opportunity of students to learn (OTL) the subject matter which is tested. The test- and OTL-data from the Second International Mathematics Study show that the implemented mathematics curricula differ within and between countries. Some groups of countries with comparable curricula could be found. By using OTL- and test-data simultaneously, we made a first step towards the identification of potential problem areas in which curriculum-developers and teacher trainers could take a closer look in order to improve the quality and outcomes of education. Of course more work has to be done to find out how powerful the OTL-measures are and especially how much they enhance the process of interpretation of the data. In this respect we consider it especially useful in further analyses to compare certain subgroups (e.g. top 10% vs. bottom 10%, boys vs girls, etc.) of students between countries after controlling for OTL.

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Husen, T. (Ed.), International Study of Achievement in Mathematics. Stockholm: Almqvist and Wiksell/New York: John Wiley and Sons, 1967.

Steiner, H.G. Comparative studies of mathematics curricula, change and stability 1960-1980. Bielefeld: Institut fur Didaktik der Mathematik der Universitat Bielefeld, 1980.

APPENDIX I
TEST ITEMS

1. $2 \text{ metres} + 3 \text{ millimetres}$ is equal to

- A 2.0003 metres
- B 2.003 metres
- C 2.03 metres
- D 2.3 metres
- E 5 metres

2. $\frac{1}{5}$ is equal to

- A 0.20%
- B 2%
- C 5%
- D 20%
- E 25%

3. If $5x + 4 = 4x - 31$, then x is equal to

- A -35
- B -27
- C 3
- D 27
- E 35

4. Four 1-litre bowls of ice cream were set out at a party. After the party, 1 bowl was empty, 2 were half full, and 1 was three-quarters full. How many litres of ice cream had been EATEN?

A $3\frac{3}{4}$

B $2\frac{3}{4}$

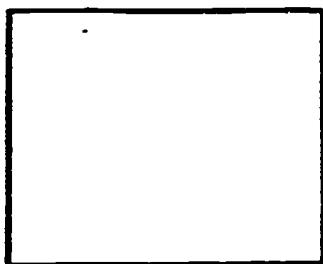
C $2\frac{1}{2}$

D $1\frac{3}{4}$

E None of these

5.

8.8 m



6.9 m

Which of the following is the closest approximation to the area of the rectangle with measurements given?

A 48 m^2

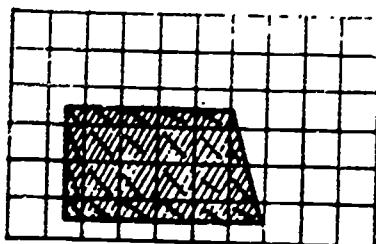
B 54 m^2

C 56 m^2

D 63 m^2

E 72 m^2

6.

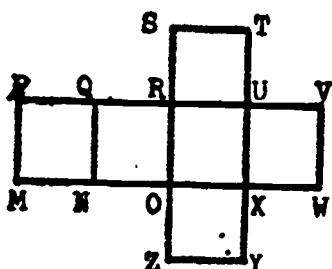


1 square unit

The area of the shaded figure, to the nearest square unit, is

- A 23 square units
- B 20 square units
- C 18 square units
- D 15 square units
- E 12 square units

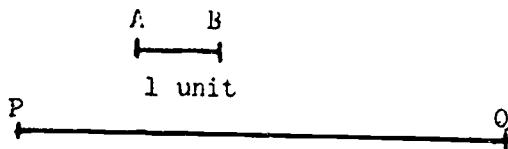
7.



The diagram shows a cardboard cube which has been cut along some edges and folded out flat. If it is folded to again make the cube, which two corners will touch P?

- A corners Q and S
- B corners T and V
- C corners W and Y
- D corners T and V
- E corners U and Y

8.



The length of \overline{AB} is 1 unit.

Which is the best estimate for the length of \overline{PQ} ?

- A 2 units
- B 6 units
- C 10 units
- D 14 units
- E 18 units

9.



On the scale the reading indicated by the arrow is between

- A 51 and 52
- B 57 and 58
- C 60 and 62
- D 62 and 64
- E 64 and 66

10. A solid plastic cube with edges 1 centimetre long weighs 1 gram. How much will a solid cube of the same plastic weigh if each edge is 2 centimetres long?

- A 8 grams
- B 4 grams
- C 3 grams
- D 2 grams
- E 1 gram

11. On a number line two points **A** and **B** are given. The point **A** is -3 and the point **B** is +7. What is the point **C**, if **B** is the midpoint of the line segment **AC**?

- A - 13
- B $-\frac{1}{2}$
- C + .2
- D + 12
- E + 17

12. A painter is to mix green and yellow paint in the ratio of 4 to 7 to obtain the colour he wants. If he has 28 litres of green paint, how many litres of yellow paint should be added?

- A 11
- B 16
- C 28
- D 49
- E 196

13. If $P = LW$ and if $P = 12$ and $L = 3$, then W is equal to

- A $\frac{3}{4}$
- B 3
- C 4
- D 12
- E 36

14. A model boat is built to scale so that it is $\frac{1}{10}$ as long as the original boat. If the width of the original boat is 4 metres, the width of the model should be

- A 0.1 metres
- B 0.4 metres
- C 1 metre
- D 4 metres
- E 10 metres

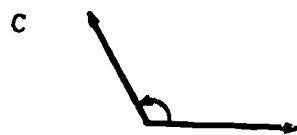
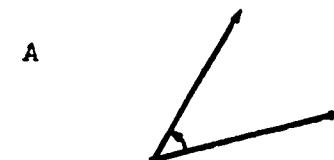
15. The value of 0.2131×0.02958 is approximately

- A 0.6
- B 0.06
- C 0.006
- D 0.0006
- E 0.00006

15. $(-2) \times (-3)$ is equal to

- A -6
- B -5
- C -1
- D 5
- E 6

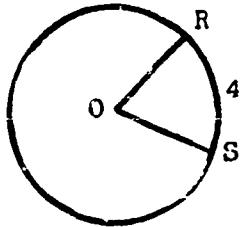
17. Which of the indicated angles is ACUTE?



18. If $\frac{4x}{12} = 0$, then x is equal to

- A 0
- B 3
- C 8
- D 12
- E 16

19.



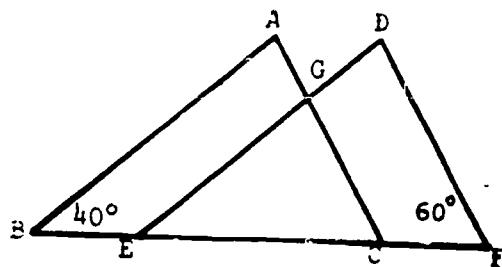
The length of the circumference of the circle with centre at O is 24 and the length of arc RS is 4. What is the size in degrees of the central angle ROS?

- A 24
- B 30
- C 45
- D 60
- E 90

20. In a discus -throwing competition, the winning throw was 61.60 metres. The second place throw was 59.72 metres. How much longer was the winning throw than the second place throw?

- A 1.12 metres
- B 1.88 metres
- C 1.92 metres
- D 2.12 metres
- E 121.32 metres

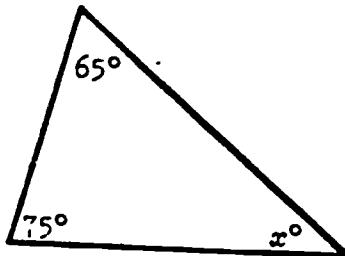
21.



In the above diagram, triangles ABC and DEF are congruent, with $\overline{BC} = \overline{EF}$.
 What is the size of angle EGC?

- A 20°
- B 40°
- C 60°
- D 80°
- E 100°

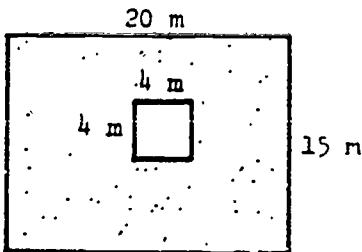
22.



x is equal to

- A 75
- B 70
- C 65
- D 60
- E 40

23.



A square is removed from the rectangle as shown. What is the area of the remaining part?

- A 316 m^2
- B 300 m^2
- C 284 m^2
- D 80 m^2
- E 16 m^2

24. Cloth is sold by the square metre. If 6 square metres of cloth cost \$4.80, the cost of 16 square metres will be

- A \$12.80
- B \$14.40
- C \$28.80
- D \$52.80
- E \$128

25. The air temperature at the foot of a mountain is 31 degrees. On top of the mountain the temperature is -7 degrees. How much warmer is the air at the foot of the mountain?

- A -38 degrees
- B -24 degrees
- C 7 degrees
- D 24 degrees
- E 38 degrees

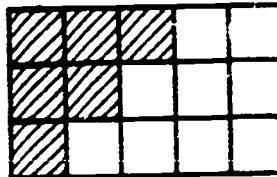
26. 0.40×6.38 is equal to

- A 0.2552
- B 2.452
- C 2.552
- D 24.52
- E 25.52

27. A shopkeeper has x kg of tea in stock. He sells 15 kg and then receives a new lot weighing $2y$ kg. What weight of tea in kg does he now have?

- A $x - 15 - 2y$
- B $x + 15 + 2y$
- C $x - 15 + 2y$
- D $x + 15 - 2y$
- E None of these

28.



In the figure the little squares are all the same size and the area of the whole rectangle is equal to 1. The area of the shaded part is equal to

- A $\frac{2}{15}$
- B $\frac{1}{3}$
- C $\frac{2}{5}$
- D $\frac{3}{8}$
- E $\frac{1}{2}$

29. The distance between two towns is usually measured in

- A millimetres
- B centimetres
- C decimetres
- D metres
- E kilometres

30. The table below gives the relation between the height from which a ball is dropped (d) and the height to which it bounces (b).

d	50	80	100	150
b	25	40	50	75

Which formula describes this relation?

- A $b = d^2$
- B $b = 2d$
- C $b = \frac{d}{2}$
- D $b = d + 25$
- E $b = d - 25$

31. $\frac{2}{5} + \frac{3}{8}$ is equal to

A $\frac{5}{13}$

B $\frac{5}{40}$

C $\frac{6}{40}$

D $\frac{16}{15}$

E $\frac{31}{40}$

32. $7\frac{3}{20}$ is equal to

A 7.03

B 7.15

C 7.23

D 7.3

E 7.6

33. In a school of 800 pupils, 300 are boys. The ratio of the number of boys to the number of girls is

A 3 : 8

B 5 : 8

C 3 : 11

D 5 : 3

E 3 : 5

34. What is 20 as a percent of 80?

- A 4%
- B 20%
- C 25%
- D 40%
- E None of these

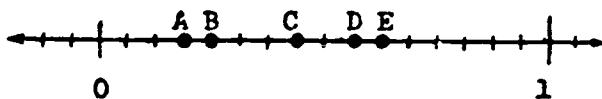
35. The sentence "a number x decreased by 6 is less than 12" can be written as the inequation

- A $x - 6 > 12$
- B $x - 6 \geq 12$
- C $x - 6 < 12$
- D $6 - x \geq 12$
- E $6 - x < 12$

36. 30 is 75% of what number?

- A 40
- B 90
- C 105
- D 225
- E 2250

37. Which of the points A, B, C, D, E on this number line corresponds to $\frac{5}{8}$?

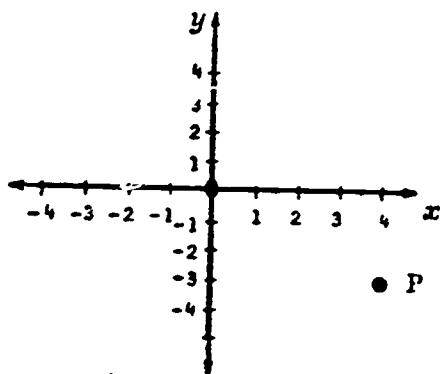


- A point A
- B point B
- C point C
- D point D
- E point E

38. 20% of 125 is equal to

- A 6.25
- B 12.50
- C 15
- D 25
- E 50

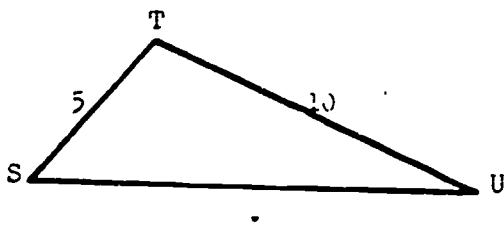
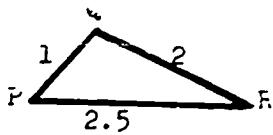
39.



What are the co-ordinates of P?

- A $(-3, 4)$
- B $(-4, -3)$
- C $(3, 4)$
- D $(4, -3)$
- E $(-4, 3)$

40.



Triangles PQR and STU are similar. How long is \overline{SU} ?

- A 5
- B 10
- C 12.5
- D 15
- E 25

41. Which of the following is equal to a quarter of a million?

A 25 250

B 40 000

C $\frac{1}{4\ 000\ 000}$

D 250 000

E 2 500 000

42. In which diagram below is the second figure the image of the first figure under a reflection in a line?

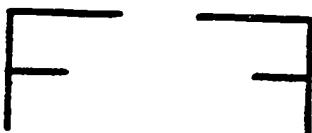
A



B



C



D



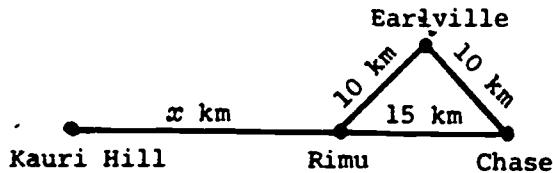
E



43. Which is the closest estimate for the answer to $5\frac{3}{7} + 6\frac{5}{8}$?

A about 8
 B about 11
 C about 12
 D about 15
 E about 31

44.



The Davis family took a car trip from Kauri Hill through Rimu to Chase. They then drove back to Rimu through Earlville, and then returned to their home in Kauri Hill. If the total distance they drove was 115 kilometres, how far is it from Kauri Hill to Rimu?

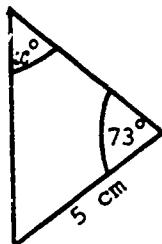
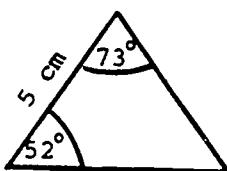
A 20 kilometres
 B 35 kilometres
 C 40 kilometres
 D 75 kilometres
 E 80 kilometres

45.

A number x is multiplied by itself and the result is added to four times the original number. This can be expressed as

A $x^2 + 4$
 B $x + 4$
 C $2x + 4$
 D $x(x^2 + 4)$
 E $x^2 + 4x$

46.



The triangles shown above are congruent. What is x ?

A 52

B 55

C 65

D 73

E 75

47. A 15 centimetre piece is cut from a ribbon 1 metre long.

What is the length of the remaining piece?

A 85 cm

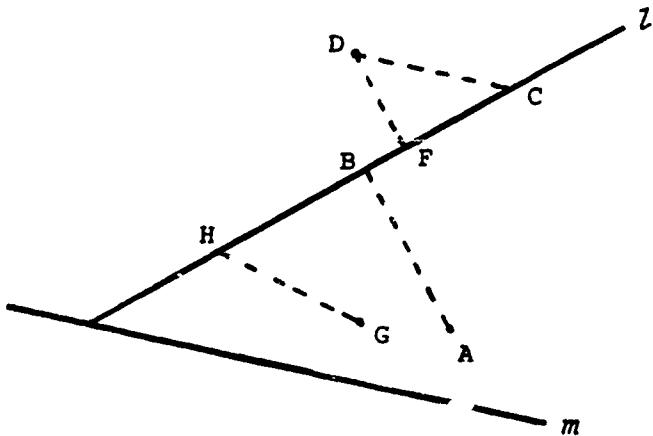
B 115 cm

C 985 cm

D 1015 cm

E 9985 cm

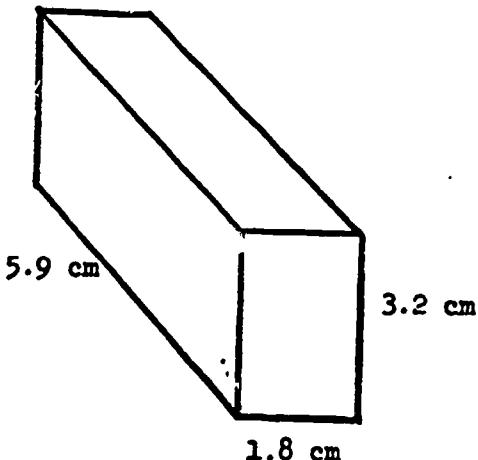
48.



If m is the direction of projection and l is the axis of projection, which of the following statements is correct?

- A $p(A) = B$
- B $p(D) = C$
- C $p(D) = F$
- D $p(G) = H$
- E $p(C) = D$

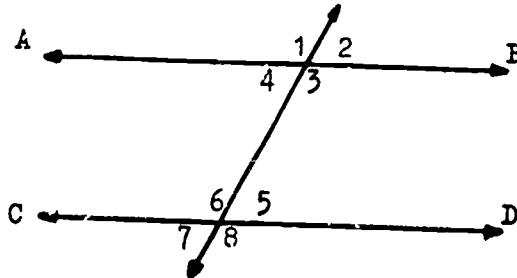
49.



The figure above shows a rectangular box. Which of the following is closest to the volume of this box?

- A 16 cm^3
- B 18 cm^3
- C 28 cm^3
- D 36 cm^3
- E 48 cm^3

50.



Lines \overleftrightarrow{AB} and \overleftrightarrow{CD} are parallel. Two angles which add up to 180° are

A angles 1 and 3

B angles 4 and 6

C angles 2 and 5

D angles 2 and 7

E angles 1 and 8

51.

A team scores an average of 3 points per game over 5 games. How many points altogether were scored in the 5 games?

A $\frac{3}{5}$

B $\frac{5}{3}$

C 3

D 5

E 15

52.

Test Score	Tally	Frequency
4	/	1
5	///	3
6	XXXX /	6
7	///	2
8	///	4
9	///	3
10	/	1

The table shows scores for a class on a 10-point test. How many in the class made a score GREATER than 7?

A 2

B 8

C 10

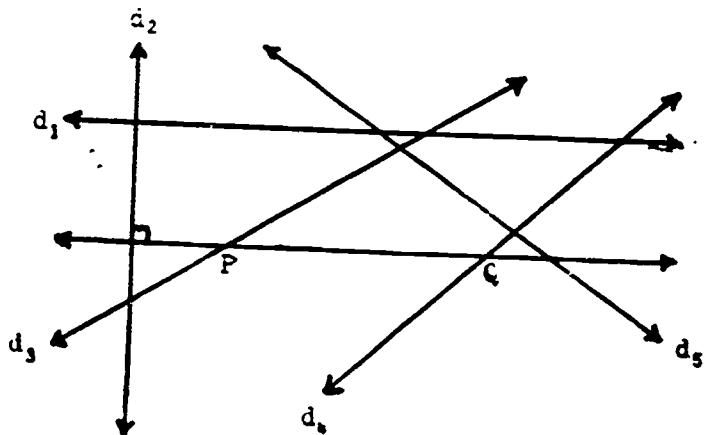
D 12

E 20

53. $\frac{3}{8} - \frac{1}{5}$ is equal to

A $\frac{1}{20}$
 B $\frac{7}{40}$
 C $\frac{7}{20}$
 D $\frac{19}{40}$
 E $\frac{2}{3}$

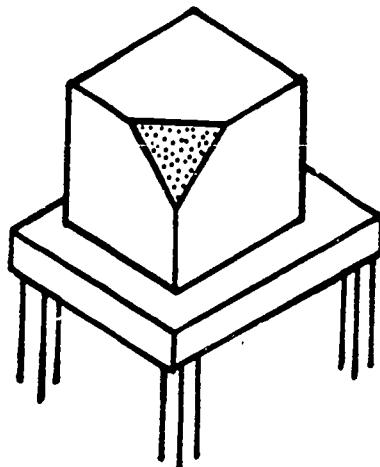
54.



Which of the lines d_1 , d_2 , d_3 , d_4 , d_5 ,
 has no point equidistant from P and Q.

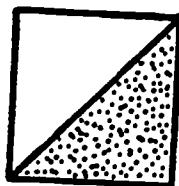
A d_1
 B d_2
 C d_3
 D d_4
 E d_5

55.

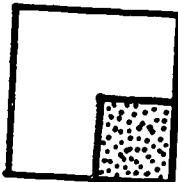


The figure above shows a wooden cube with one corner cut off and shaded. Which of the following drawings shows how this cube would look when viewed from directly above it.

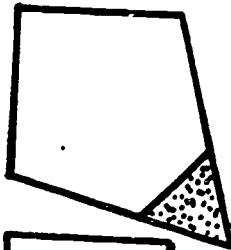
A



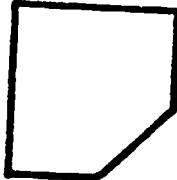
B



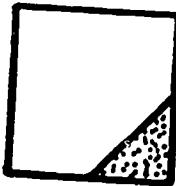
C



D



E



56.

<i>x</i>	3	6	<i>P</i>
<i>y</i>	7	<i>Q</i>	35

The table above shows the values of *x* and *y*, where *x* is proportional to *y*. What are the values of *P* and *Q*?

- A $P = 14$ and $Q = 31$
- B $P = 10$ and $Q = 14$
- C $P = 10$ and $Q = 31$
- D $P = 14$ and $Q = 15$
- E $P = 15$ and $Q = 14$

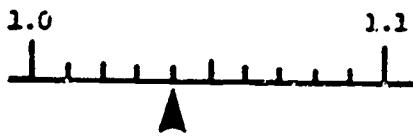
57.

- 1st row 1
- 2nd row $1 - 1$
- 3rd row $1 - 1 + 1$
- 4th row $1 - 1 + 1 - 1$
- 5th row $1 - 1 + 1 - 1 + 1$

What is the sum of the 50th row?

- A 0
- B 1
- C 2
- D 25
- E 30

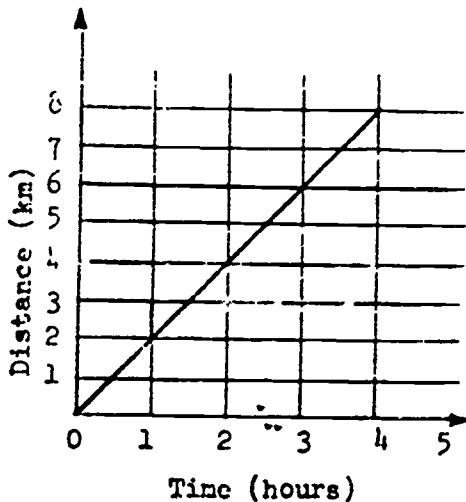
58.



The position on the scale indicated by the arrow is

- A 1.004
- B 1.04
- C 1.08
- D 1.4
- E 1.8

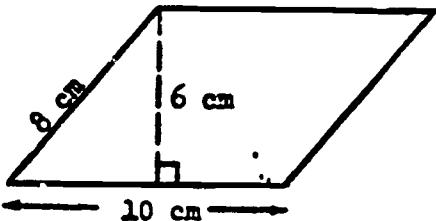
59.



The graph shows the distance travelled by a tractor during a period of 4 hours. How fast is the tractor moving?

- A 1 kilometre per hour
- B 2 kilometres per hour
- C 4 kilometres per hour
- D 8 kilometres per hour
- E There is not enough information

60.



What is the area of the parallelogram?

- A 30 cm^2
- B 36 cm^2
- C 48 cm^2
- D 60 cm^2
- E 80 cm^2

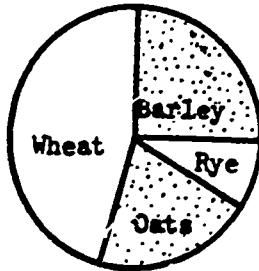
61.

4:1
0.004 24.56

In the division above, the correct answer is

- A 0.614
- B 6.14
- C 61.4
- D 614
- E 6140

62.



The circle graph shows the proportions of various grain crops produced by a country.
Which of the following statements is TRUE?

- A More oats than rye is produced.
- B The largest crop is barley.
- C Equal quantities of wheat and barley are produced.
- D The smallest crop is oats.
- E Wheat and oats together make up less than half the total grain crop.

63. The price of an article was \$100.
 The price was first raised by 10% and was then reduced
 by 10% of the new price. What is the price of the
 article now?

- A \$90
- B \$99
- C \$100
- D \$101
- E \$110

64. If $10^2 \times 10^3 = 10^n$ then n is equal to

- A 4
- B 5
- C 6
- D 8
- E 9

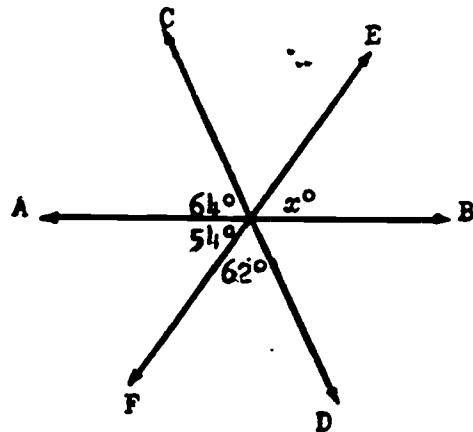
65. A car takes 15 minutes to travel 10 kilometres.
 What is the speed of the car?

- A 30 kilometres per hour
- B 40 kilometres per hour
- C 60 kilometres per hour
- D 90 kilometres per hour
- E 150 kilometres per hour

66. If $x = -3$, the value of $-3x$ is

- A -9
- B -6
- C -1
- D 1
- E 9

67.



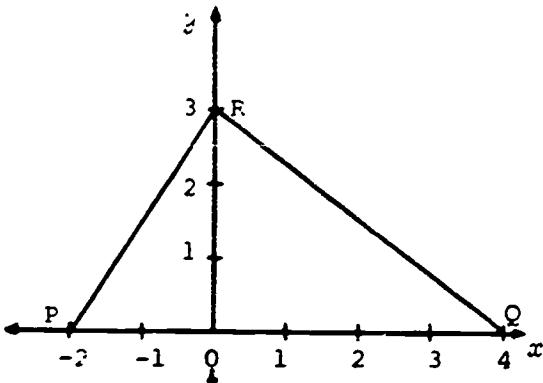
AB, CD, and EF are intersecting straight lines as shown above. The sizes of certain angles are shown. x is equal to

- A 54
- B 62
- C 64
- D 126
- E 128

68. When $x = 2$, $\frac{7x + 4}{5x - 4}$ is equal to

- A 11
- B 3
- C $\frac{11}{5}$
- D $\frac{9}{5}$
- E $\frac{7}{5}$

69.



What is the area of triangle PQR?

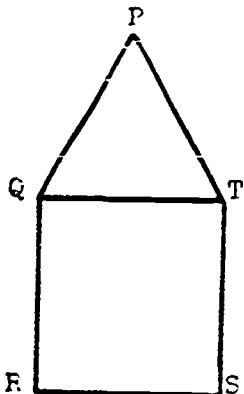
- A 3 square units
- B 6 square units
- C 9 square units
- D 12 square units
- E 18 square units

70.

What is the square root of 12×75 ?

- A 6.25
- B 30
- C 87
- D 625
- E 900

71.



The figure $QRST$ is a square and PQT an equilateral triangle. If $PQ = 6$ cm then the area of the square is

- A 64 cm^2
- B 48 cm^2
- C 40 cm^2
- D 36 cm^2
- E 24 cm^2

72.

Peter and Paul decided to start saving money. Peter can save 3 dollars each month and Paul can save 5 dollars. At this rate, after how many months will Paul have exactly 10 dollars more than Peter?

- A 2
- B 3
- C 4
- D 5
- E 8

73. 0.00046 is equal to

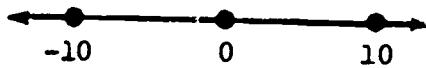
- A 46×10^{-3}
- B 4.6×10^{-4}
- C 0.46×10^3
- D 4.6×10^4
- E 46×10^5

74.

One of the following points can be joined to the point $(-3,4)$ by a line segment which cuts NEITHER the x NOR the y axis. Which one?

- A $(-2,3)$
- B $(2,-3)$
- C $(2,3)$
- D $(-2,-3)$
- E $(4,-3)$

75.



Which of the following sequences of numbers is in the order in which they occur from left to right on the number line?

A $0, \frac{1}{2}, -1$

B $0, -1, \frac{1}{2}$

C $-1, -\frac{1}{2}, 0$

D $-1, 0, -\frac{1}{2}$

E $-\frac{1}{2}, -1, 0$

76. 72 is equal to

- A 7200
- B 720
- C 72
- D 7.2
- E 0.72

77. Which of the following is thirty-seven thousandths?

- A 37 000
- B 37
- C 0.37
- D 0.037
- E 0.0037

78. The petals on 100 flowers of different kinds were carefully counted, and the results are shown in this table.

No. of petals	Frequency
10-12	5
13-15	22
16-18	48
19-21	18
22-24	7

How many of the flowers had fewer than 19 petals?

- A 48
- B 52
- C 73
- D 75
- E 93

79. There are 7 000 000 girls under the age of 21 in a country with a total population of 36 000 000. If a circle graph (pie chart) were drawn showing the distribution of the population, the size of the angle in the sector representing girls under the age of 21 would be

A 7°
 B 20°
 C 21°
 D 70°
 E 72°

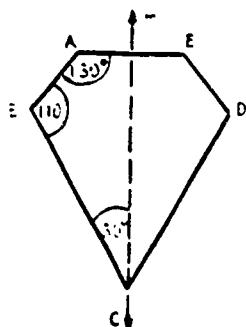
80.

m	-1	1	2	4
n	-1	3	5	9

For the table shown, a formula that relates m and n is

A $n = m$
 B $r = 3m$
 C $r = -m^2 + 1$
 D $r = m^2 + 1$
 E $n = 2m + 1$

81.



The line m is a line of symmetry for figure ABCDE. The size of angle BCD is

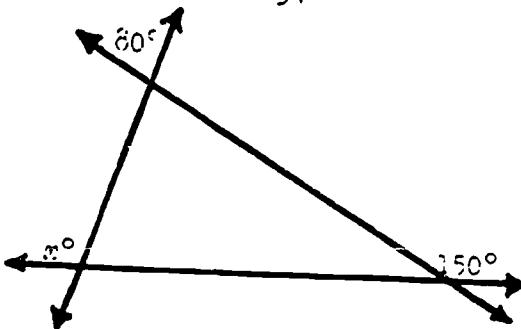
- A 30°
- B 50°
- C 60°
- D 70°
- E 130°

82. Rosemarie walked from Riverview to Bridgeport, which are 3.1 kilometres apart. During her walk she lost her watch, went back 1.7 kilometres to find it, and then continued in the original direction until she reached Bridgeport. How many kilometres had Rosemarie walked altogether when she arrived at Bridgeport?

- A 1.4
- B 4.8
- C 6.5
- D 8.2
- E None of these

83.

54



Three straight lines intersect as shown in the diagram. What is x equal to?

- A 30
- B 50
- C 60
- D 110
- E 150

84. Joe had three test scores of 78, 76 and 74, while Mary had scores of 72, 82, and 74. How did Joe's average compare with Mary's?

- A Joe's was 1 point higher
- B Joe's was 1 point lower
- C Both averages were the same
- D Joe's was 2 points higher
- E Joe's was 2 points lower

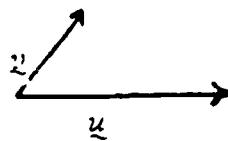
85.

$$\frac{\frac{3}{5}}{\frac{2}{7}} \text{ is equal to}$$

- A $\frac{21}{10}$
- B $\frac{5}{12}$
- C $\frac{10}{21}$
- D $\frac{6}{35}$
- E $\frac{31}{35}$

56

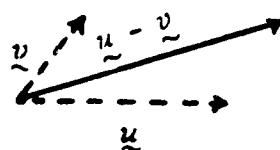
86.



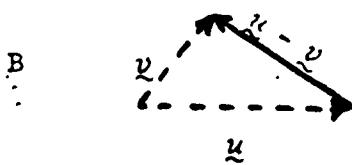
\underline{u} and \underline{v} are two vectors.

Which figure below represents $\underline{u} - \underline{v}$?

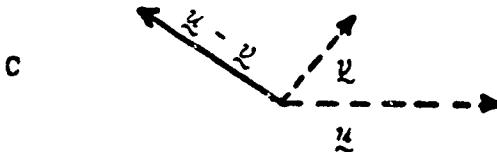
A



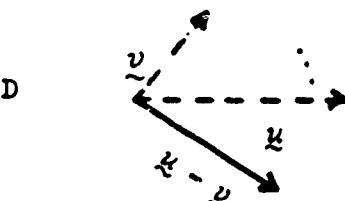
B



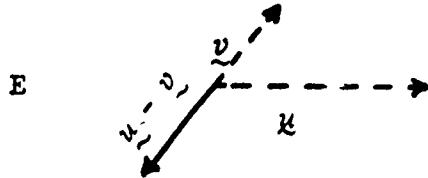
C



D



E

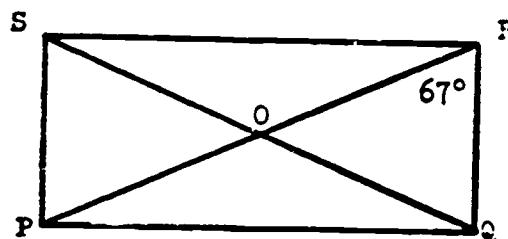


87. $6x - 3 = 15$
 $\Rightarrow 6x = 15 + 3 \quad (i)$
 $\Rightarrow 6x = 12 \quad (ii)$
 $\Rightarrow x = \frac{12}{6} \quad (iii)$
 $\Rightarrow x = 2 \quad (iv)$

The first error in the above reasoning, if one exists, FIRST APPEARS in line

- A (i)
- B (ii)
- C (iii)
- D (iv)
- E None of these, there is no error.

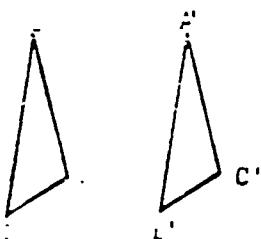
88.



If the above rectangle the size of angle ROQ is

- A 23°
- B 45°
- C 46°
- D 54°
- E 67°

89.



△ABC and △A'B'C' are congruent and their corresponding sides are parallel. △ABC maps onto △A'B'C' by a

C

- A reflection
- B glide reflection
- C rotation
- D enlargement
- E translation

90. Which of the following operations with whole numbers will **ALWAYS** give a whole number?

C

Σ

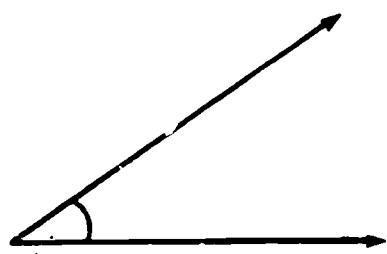
- I Addition
- II Multiplication
- III Division

- A I only
- B II only
- C III only
- D I and II only
- E II and III only

91. The value of $2^3 \times 3^2$ is

- A 30
- B 36
- C 64
- D 72
- E None of these

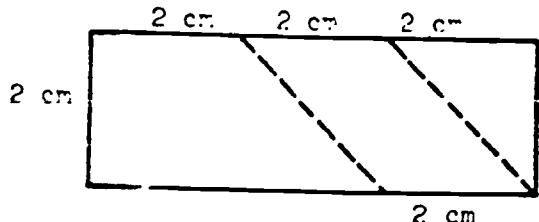
92.



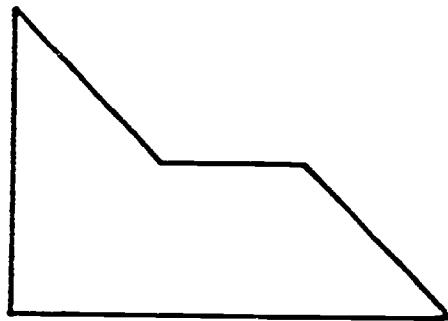
The size of the angle shown is nearest to:

- A 155°
- B 145°
- C 50°
- D 35°
- E 15°

93.



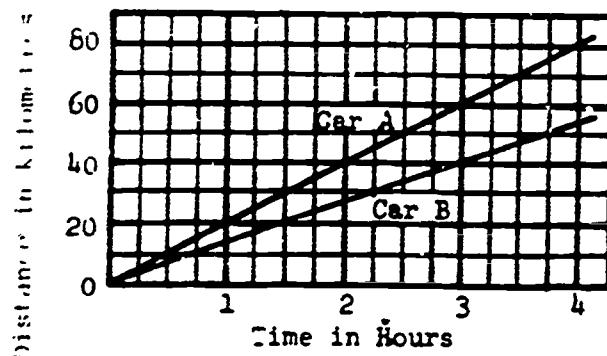
The rectangle shown above is cut along the dotted lines and the three parts put together, without overlapping, to give the figure shown below.



The area in square centimetres of this figure is

- A 8 cm^2
- B 10 cm^2
- C 12 cm^2
- D 14 cm^2
- E 16 cm^2

94



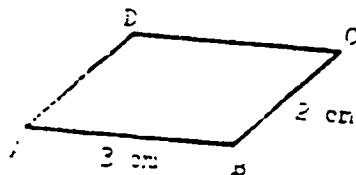
How much longer does it take for car B to go 50 kilometres than it does for car A to go 50 kilometres?

- A 1 hour 15 minutes
- B 1 hour 30 minutes
- C 2 hours
- D 2 hours 30 minutes
- E 2 hours 35 minutes

95. Which of these numbers is a prime number?

- A 21
- B 22
- C 23
- D 24
- E 25

96



\overline{AB} is parallel to \overline{DC} and \overline{AD} is parallel to \overline{BC}
 Quadrilateral ABCD is a

- A rhombus
- B parallelogram
- C square
- D rectangle
- E none of these

97. $N = 10^3 + 10^1 + 10^0 + 10^{-2}$

N is equal to

- A $N = 0$
- B $N = 20$
- C $N = 1011.01$
- D $N = 100$
- E None of these

98. If there are 300 calories in 100 grams of a certain food, how many calories are there in a 30 gram portion of that food?

- A 90
- B 100
- C 900
- D 1000
- E 9000

99. $\therefore \text{Ans}(115)$ $5x + 3x + 2x = 40$

A $7x + 3x$

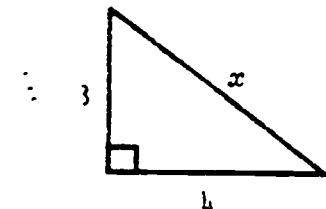
B $8x + 2x$

C $6x$

D $7x - 1$

E $7x + y$

100.



Which of these is a correct statement for this triangle?

A $x^2 = 3^2 + 4^2$

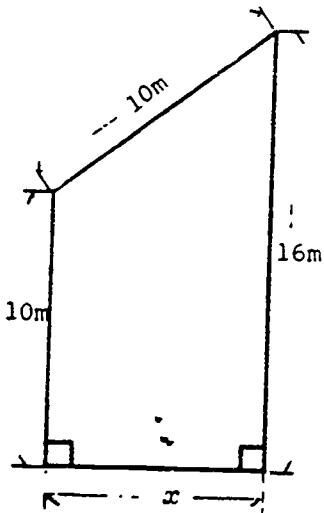
B $x^2 + 3^2 = 4^2$

C $x = 4^2 - 3^2$

D $x^2 = 4^2 - 3^2$

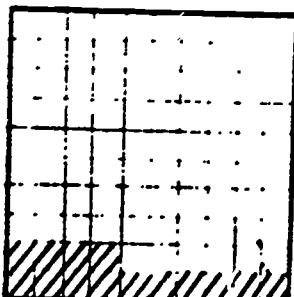
E $x = 4 + 3$

101.

*x* is equal to

- A 4 m
- B 6 m
- C 8 m
- D 10 m
- E 12 m

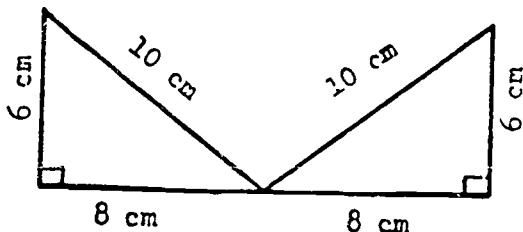
102.



The large square has area 1 square unit. The area of the shaded part is

- A 14 square units
- B 1.4 square units
- C 0.14 square units
- D 0.014 square units
- E 0.0014 square units

103.



The total area of the two triangles is

A $6 \times 8 \text{ cm}^2$

B $\frac{6 \times 8}{2} \text{ cm}^2$

C $\frac{10 \times 6}{2} \text{ cm}^2$

D $\frac{16 \times 12}{2} \text{ cm}^2$

E $\frac{20 \times 12}{2} \text{ cm}^2$

104. If y dollars are shared equally among four boys, how many dollars does each boy receive?

A $\frac{y}{4}$

B $\frac{4}{y}$

C 4

D $\frac{y}{4}$

E $4y$

105. $(-6) - (-8)$ is equal to

A 14

B 2

C -2

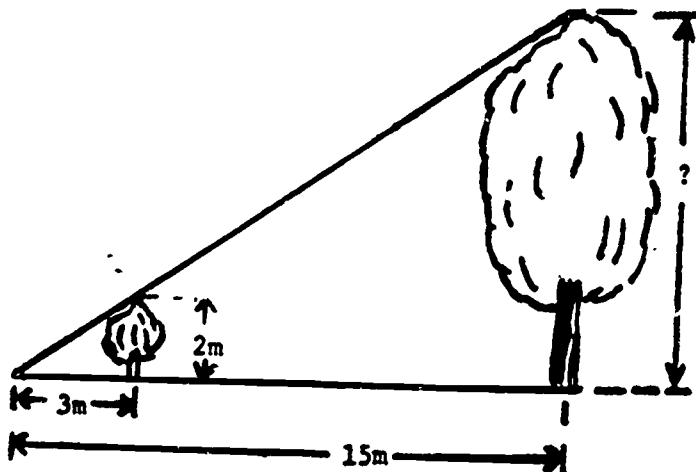
D -10

E -14

106. The length of a box was measured and found to be 9 centimetres TO THE NEAREST CENTIMETRE. Which of these could have been the length of the box measured more accurately?

- A 10 cm
- B 9.9 cm
- C 9.62 cm
- D 9.6 cm
- E 8.6 cm

107.



The picture above shows how Pedro used a short tree to find the height of the tall tree. What answer should Pedro get?

- A 10 metres
- B 12 metres
- C 14 metres
- D 17 metres
- E 20 metres

108. $\sqrt{75}$ is between

- A 4 and 5
- B 5 and 6
- C 6 and 7
- D 7 and 8
- E 8 and 9

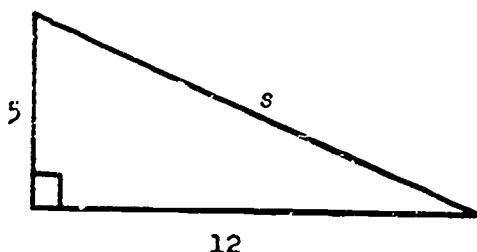
109. $(22 \times 18) - (47 + 59)$ is equal to

- A 290
- B 300
- C 384
- D 408
- E 502

110. There are 35 students in a class. $\frac{1}{5}$ of them come to school by bus, another $\frac{2}{5}$ come by bicycle. How many come to school by other means?

- A 7
- B 14
- C 21
- D 28
- E 35

111.



What is the value of s ?

- A 7
- B 13
- C 15
- D 17
- E None of these

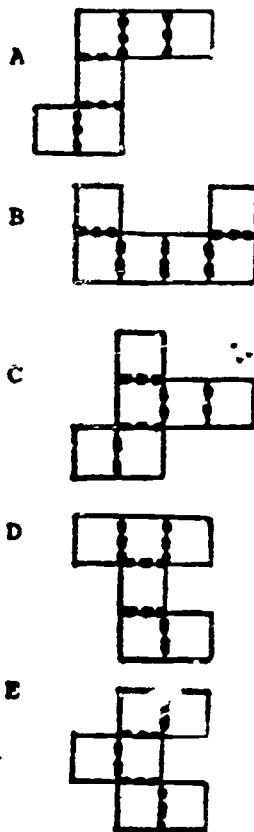
112. Which of the following is most likely to be nearest to the weight of a normal man?

- A 8.5 kg
- B 65 kg
- C 185 kg
- D 850 kg
- E 1850 kg

113. Which of the following is a pair of equivalent fractions?

- A $\frac{5}{8}$ and $\frac{2}{3}$
- B $\frac{5}{6}$ and $\frac{2}{3}$
- C $\frac{4}{5}$ and $\frac{14}{15}$
- D $\frac{7}{5}$ and $\frac{9}{15}$
- E $\frac{1}{2}$ and $\frac{14}{24}$

114. Which of the following patterns can be folded along the dotted sides to make a cube?



115. $1\frac{2}{5} - \frac{1}{2}$ is equal to

A $\frac{2}{3}$

B $\frac{9}{10}$

C $1\frac{1}{10}$

D $1\frac{1}{7}$

E $1\frac{1}{3}$

116. There are five black buttons and one red button in a jar. If you pull out one button at random, what is the probability that you will get the red button.

A 0

B $\frac{1}{6}$

C $\frac{1}{5}$

D $\frac{5}{6}$

E 1

117. You wish to know whether SLOSH is the most popular soft-drink in your school. The way of finding out, from among the following, which will give results you can be most sure of, will be to

A note the number of empty SLOSH bottles in the rubbish bins.

B ask the manager of the snack bar how many cases of SLOSH he has ordered in the last month.

C ask your friends whether they think that SLOSH is the most popular soft-drink.

D discuss with the driver of the soft-drink delivery truck what he thinks of SLOSH.

E keep a record of soft-drink sales in the school by brand name over a period of 1 week.

118.

A group of children was divided into 7 teams with nine in each team. Later, the same group of children was divided into teams with seven in each team. How many teams were there then?

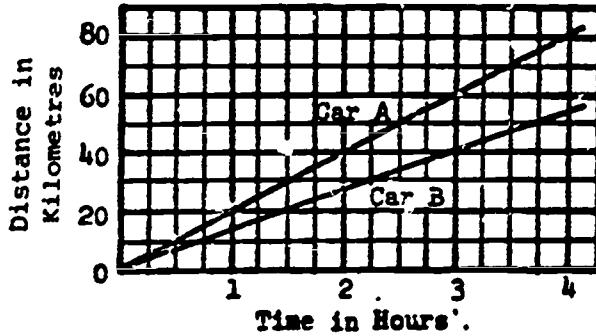
- A 7
- B 8
- C 9
- D 16
- E 63

119.

If two triangles are SIMILAR, which of the following statements is TRUE?

- A Their corresponding angles MUST be equal.
- B Their corresponding sides MUST be equal.
- C Their corresponding sides MUST be parallel.
- D They MUST have the same area.
- E They MUST have the same shape and size.

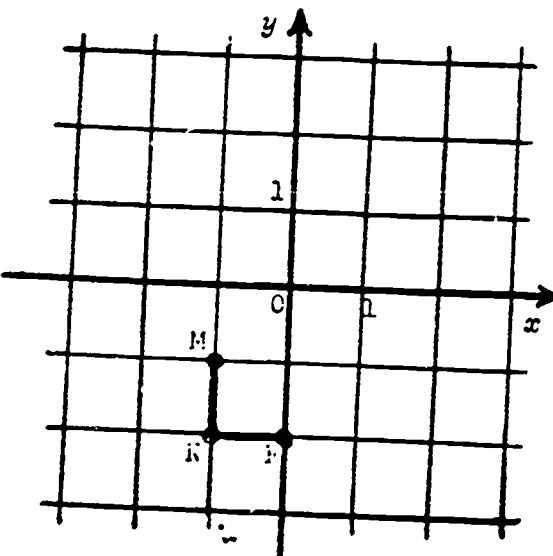
120.



Three hours after starting, car A is how many kilometres ahead of car B?

- A 2
- B 10
- C 15
- D 20
- E 25

121.



Suppose you start at point $M(-1, -1)$, move a distance of one unit to $N(-1, -2)$, then turn left and move one unit to the point $P(0, -2)$. If you again turn left and move one unit, you will now be at the point with coordinates

- A $(1, -2)$
- B $(0, -3)$
- C $(0, -1)$
- D $(-1, -2)$
- E None of the above

122. The cost of printing greeting cards consists of a fixed charge of 10 cents and a charge of 6 cents for each card printed. Which of the following equations can be used to determine the cost of printing n cards?

- A $\text{cost} = (100 + 6n)$ cents
- B $\text{cost} = (106 + n)$ cents
- C $\text{cost} = (6 + 100n)$ cents
- D $\text{cost} = (106n)$ cents
- E $\text{cost} = (60n)$ cents

123. "Six times a certain number (call it q) equals the sum of eight and twice the number." This can be written as

- A $6q = 2(8 + q)$
- B $6(q + 8) = 2q$
- C $6(q + 8) = 8 + 2q$
- D $6q = 8 + 2q$
- E $q = 1$

124. Candidate A received 70 percent of the votes cast in an election. If 4200 votes were cast in the election, how many votes did Candidate A receive?

- A 2800
- B 2900
- C 2940
- D 3000
- E 4130

125. What is the capacity of a cubic container 10 cm by 10 cm by 10 cm?

- A 1 litre
- B 10 litres
- C 100 litres
- D 1000 litres
- E 1000 centimetres

126. If $x = y = z = 1$, then $\frac{x - z}{x + y}$ is equal to

- A -2
- B -1
- C 0
- D $\frac{1}{2}$
- E 1

127. Michael has a large number of wooden blocks which are cubical in shape with each edge 1 centimetre long. What is the maximum number of these blocks that can be used to fill a rectangular box with interior dimensions 10 centimetres long, 10 centimetres wide and 7 centimetres high?

- A 27
- B 70
- C 140
- D 280
- E 700

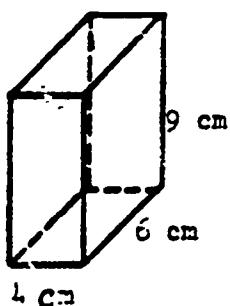
128. If the ratio of 2 to 5 equals the ratio of n to 100, then n is equal to

- A 10
- B 20
- C 40
- D 150
- E 250

129. One bell rings every 8 minutes, a second bell rings every 12 minutes. They both ring at exactly 12 o'clock. After how many minutes will they next ring together?

- A 8
- B 12
- C 20
- D 24
- E 96

130. What is the SURFACE AREA of this solid rectangular box?



- A 50 square centimetres
- B 100 square centimetres
- C 114 square centimetres
- D 216 square centimetres
- E 228 square centimetres

131. 3.23×10^5 is equal to

- A 0.0000323
- B 3.23000
- C 32 300
- D 323 000
- E 32 300 000

132. The speed of sound is approximately 340 metres per second. How long will it take before the sound of a car horn reaches your ears if the car is 714 metres away?

- A 0.21 seconds
- B 2.1 seconds
- C 21 seconds
- D 210 seconds
- E None of these

133. A quadrilateral MUST be a parallelogram if it has

- A one pair of adjacent sides equal
- B one pair of parallel sides
- C a diagonal as axis of symmetry
- D two adjacent angles equal
- E two pairs of parallel sides

134. Which of the following is FALSE when a , b , and c are different real numbers?

A $(a + b) + c = a + (b + c)$
 B $ac = ba$
 C $a + b = b + a$
 D $(abc)c = a(bc)$
 E $a - b = b - a$

135. 74.236 rounded to the nearest HUNDREDTH is

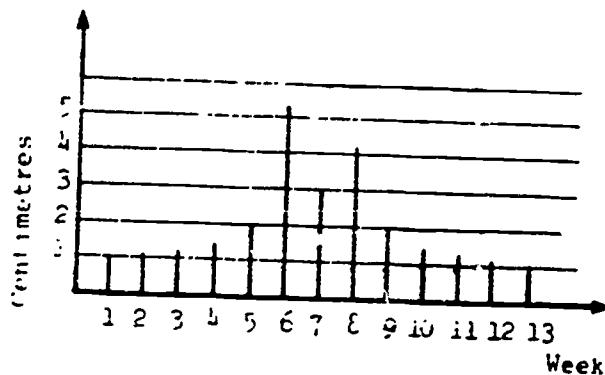
A 74.2
 B 74.3
 C 74.23
 D 74.24
 E 74.240

136. A bowling ball travels 4 metres per second. The distance ~~is~~ metres travelled in t seconds is given by $d = 4t$. In the table below x is equal to

t	d
0	0
1	4
2	8
3	x
4	16

A 6
 B 10
 C 12
 D 14
 E None of these

137.



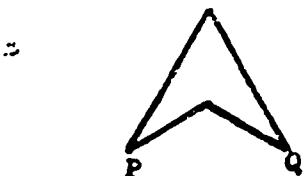
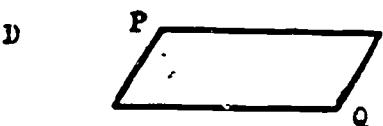
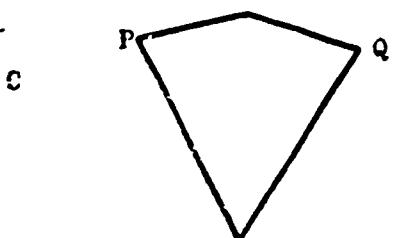
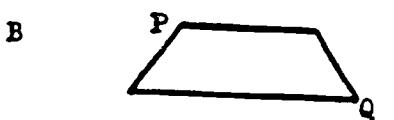
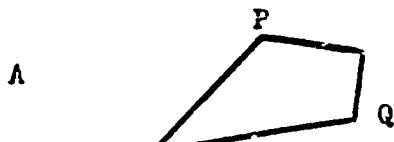
In the graph, rainfall in centimetres is plotted for 13 weeks. The average weekly rainfall during the period is approximately

- A 1 centimetre
- B 2 centimetres
- C 3 centimetres
- D 4 centimetres
- E 5 centimetres

138. 162×45 is equal to

- A 1378
- B 1458
- C 5890
- D 6290
- E 7290

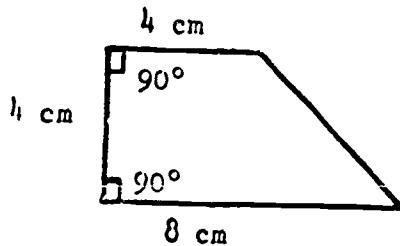
139. If segment \overline{PQ} were drawn for each figure shown below, it would divide the figure into two congruent triangles. Which figure?



140. The arithmetic mean (average) of: 1.50, 2.40, 3.75 is equal to

- A 2.40
- B 2.55
- C 3.75
- D 7.65
- E None of these

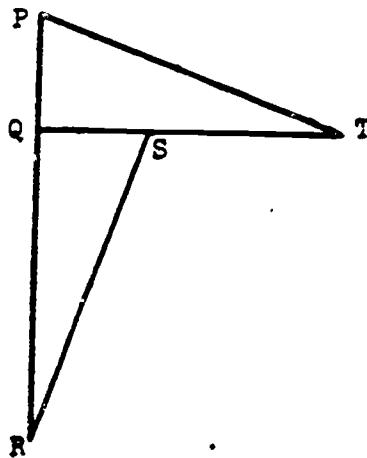
141.



There is a brass plate of the shape and dimensions shown in the figure above. What is its area in square centimetres?

- A 16
- B 24
- C 32
- D 64
- E 96

142.



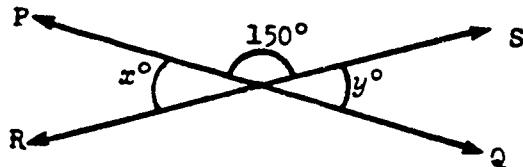
Triangle POT can be rotated onto triangle SQR. The centre of rotation is

- A Point P
- B Point Q
- C Point R
- D Point S
- E Point T

143. Since $4 \times 9 = 36$, $\sqrt{36}$ is equal to

- A 4×9
- B 4×3
- C 2×9
- D 2×3
- E $\sqrt{2} \times \sqrt{3}$

144.



If, in the given figure PQ and RS are intersecting straight lines, then $x + y$ is equal to

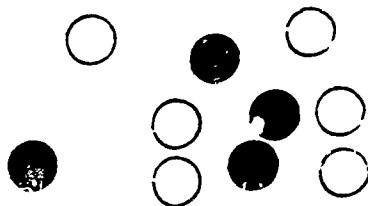
- A 15
- B 30
- C 60
- D 180
- E 300

145. $\frac{a}{15} - \frac{b}{5}$ is equal to

- A $\frac{a - 3b}{15}$
- B $\frac{5a - 15b}{15}$
- C $\frac{a-b}{10}$
- D $\frac{a-b}{75}$
- E None of these

16.

The picture shows some black and some white marbles. Of all these marbles what fraction is white?



A $\frac{1}{2}$

B $\frac{6}{4}$

C $\frac{4}{6}$

D $\frac{6}{10}$

E $\frac{4}{10}$

147.

What is the volume of a rectangular box with interior dimensions 10 cm long, 10 cm wide, and 7 cm high?

A 27 cm^3

B 70 cm^3

C 140 cm^3

D 280 cm^3

E 700 cm^3

148. A runner ran 3000 metres in exactly 8 minutes. What was his average speed in metres per second?

A 3.75

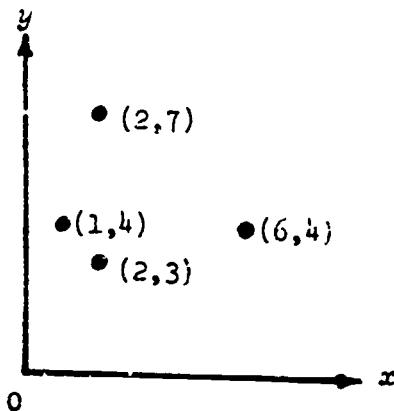
B 6.25

C 16.0

D 37.5

E 62.5

149.



The straight line joining the points $(2,3)$ and $(2,7)$ cuts the straight line joining the points $(1,4)$ and $(6,4)$ at the point

A $(4,2)$

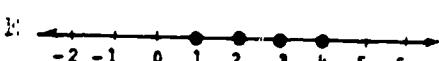
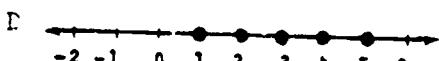
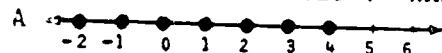
B $(1,4)$

C $(1,3)$

D $(2,3)$

E $(2,4)$

150. The set of integers less than 5 is represented on one of the number lines shown below. Which one?



151. Which of the following is (are) TRUE?

I $(53 \times 73) \times 17 = 53 \times (73 \times 17)$

II $133 \times (78 + 89) = (133 \times 78) + 89$

III $133 \times (78 + 89) = (133 \times 78) + (133 \times 89)$

A I only

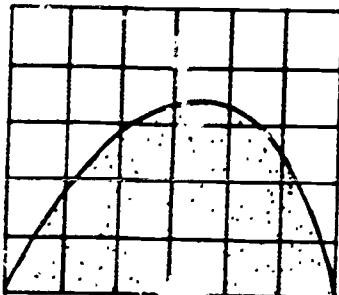
B II only

C III only

D I and II only

E I and III only

152.



Each of the small squares in the figure is 1 square unit. Which is the best estimate of the area of the shaded region?

- A 10 square units
- B 12 square units
- C 14 square units
- D 16 square units
- E 18 square units

153. Here is a table of data and a graph of the same data. What is x ?

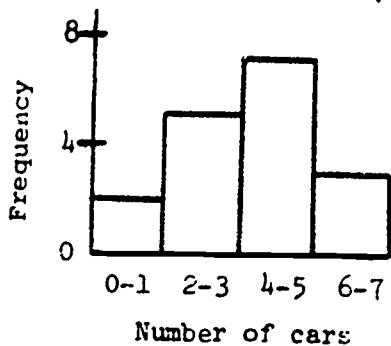
Number of Cars . Frequency

0 or 1 2

2 or 3 x

4 or 5 7

6 or 7 3



A 2

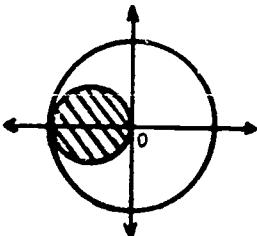
B 3

C 4

D 5

E 6

154. The area of the shaded circle is what part of the area of the large circle?



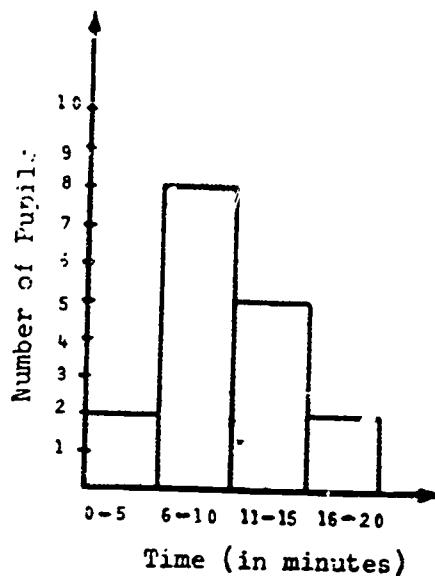
- A $\frac{1}{6}$
- B $\frac{1}{5}$
- C $\frac{1}{4}$
- D $\frac{1}{3}$
- E $\frac{1}{2}$

155. Find the sum:

$$\begin{array}{r}
 3 \text{ weeks } 5 \text{ days} \\
 + 9 \text{ weeks } 6 \text{ days} \\
 \hline
 \end{array}$$

- A 12 weeks 1 day
- B 12 weeks 4 days
- C 13 weeks 1 day
- D 13 weeks 4 days
- E 13 weeks 11 days

156.



The graph shows the time of travel by pupils from home to school. How many pupils must travel for MORE than 10 minutes?

- A 2
- B 5
- C 7
- D 8
- E 15

157. Matchsticks are arranged as follows:



If the pattern is continued, how many matchsticks are used in making the 10th figure?

- A 30
- B 33
- C 36
- D 39
- E 42

158. Subtract:

$$\begin{array}{r} 1054 \\ - 865 \\ \hline \end{array}$$

- A 189
- B 199
- C 211
- D 289
- E 299

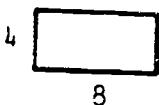
159. In a school election with three candidates, Joe received 120 votes. Mary received 50 votes, and George received 30 votes. What percent of the total number of votes did Joe receive?

- A $\frac{6}{10}\%$
- B 40%
- C 60%
- D 80%
- E 120%

160. On level ground, a boy 5 units tall cast a shadow 3 units long. At the same time a nearby telephone pole 45 units high cast a shadow the length of which, in the same units, is

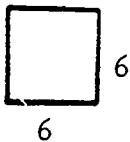
- A 24
- B 27
- C 30
- D 60
- E 75

161.

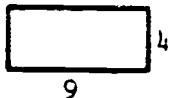


One of the following figures is congruent with the figure above. Which one?

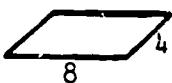
A



B



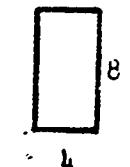
C



D

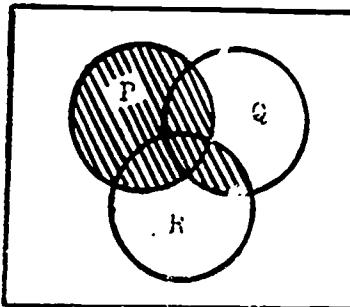


E



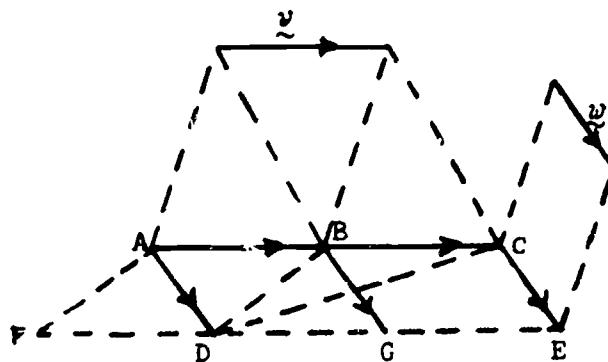
162.

The symbol $P \cap Q$ represents the intersection of sets P and Q and the symbol $P \cup Q$ represents the union of sets P and Q . Which of the following represents the shaded portion of the diagram below?



- A $(P \cap Q) \cup R$
- B $P \cup (Q \cap R)$
- C $P \cap Q \cup R$
- D $(P \cap Q) \cap R$
- E $(P \cup Q) \cap R$

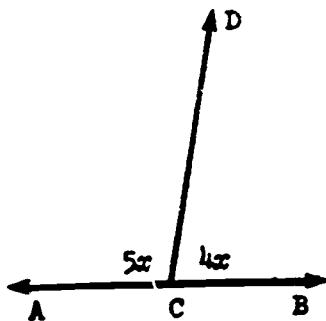
163.



Given vectors \underline{v} and \underline{w} as shown in the figure above, what is \underline{DB} , the vector from D to B?

- A $\underline{v} + \underline{w}$
- B $\underline{v} - \underline{w}$
- C $\underline{w} - \underline{v}$
- D $-\underline{w} - \underline{v}$
- E $\underline{v} + 2\underline{w}$

164.



If AB is a straight line, what is the size in degrees of angle BCD?

- A 20
- B 40
- C 50
- D 80
- E 100

165. Which equation is true for ALL values of n ?

A $2 + n = n + 2$

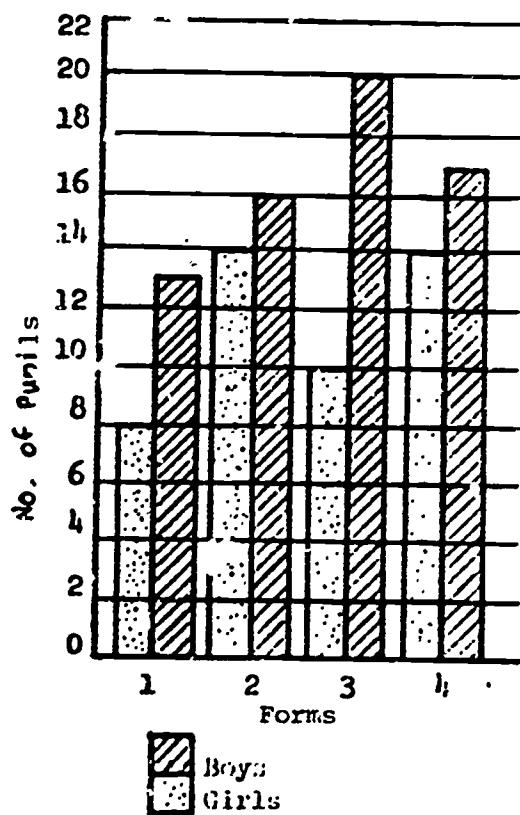
B $3 + n = 4 + 2$

C $n + 1 = 1$

D $2n + 1 = n$

E $n + 3 = 3n$

166. PUPILS IN FORMS 1, 2, 3, AND 4



Which of these is a TRUE statement about the information shown on the graph?

A Form 2 is the smallest class

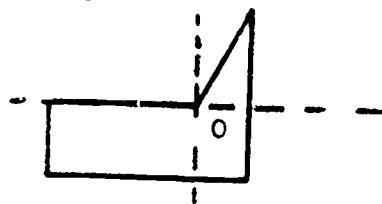
B Forms 2 and 4 have the same number of pupils

C Form 3 has twice as many boys as girls

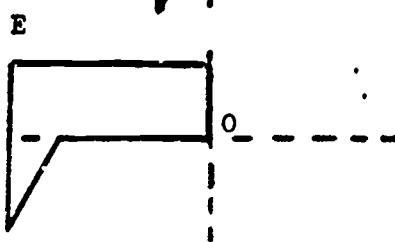
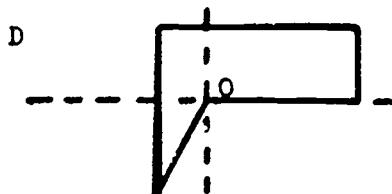
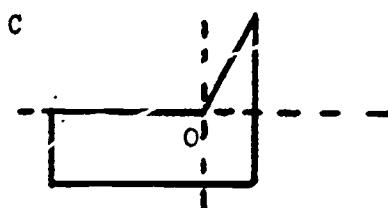
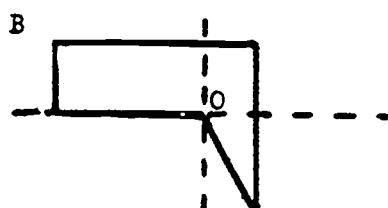
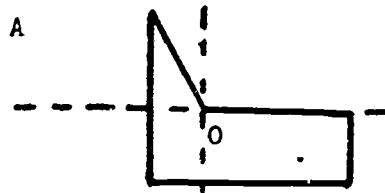
D Form 4 has more girls than boys

E Form 1 has as many boys as there are girls in Form 4

167.



A half-turn about 0 is applied to the figure above. Which of the figures below is the result?



168.

How many pieces of pipe each 20 metres long would be required to construct a pipeline 1 kilometre in length?

- A 5
- B 50
- C 500
- D 5 000
- E 50 000

169. In a quadrilateral, two of the angles are each 110° , and the third angle is 90° . What is the size of the remaining angle?

A 50°
 B 90°
 C 130°
 D 140°
 E None of the above.

170. $\frac{1}{2} \times \frac{1}{4}$ is equal to

A $\frac{1}{8}$
 B $\frac{1}{6}$
 C $\frac{2}{8}$
 D $\frac{2}{4}$
 E 8

171. $\frac{x}{2} < 7$ is equivalent to

A $x < \frac{7}{2}$
 B $x < 5$
 C $x < 14$
 D $x > 5$
 E $x > 14$

172.

Lemonade costs a cents for each bottle, but there is a refund of b cents on each empty bottle. How much will Henry have to pay for x bottles if he brings back y empties?

A $ax + by$ cents

B $ax - by$ cents

C $(a - b)x$ cents

D $(a - x) - (b + y)$ cents

E None of these

173.

Which of the following equals $7 \times (3 + 9)$?

A $(7 \times 3) + (7 \times 9)$

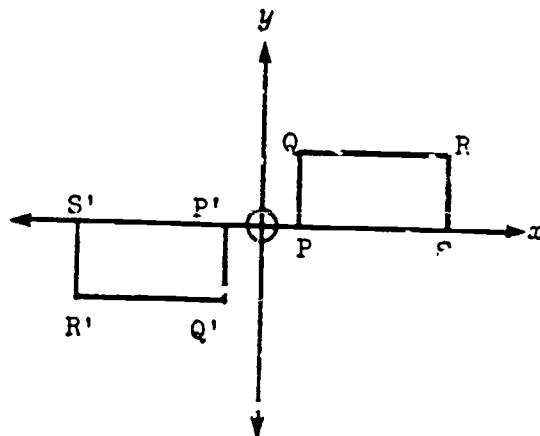
B $(7 \times 9) + (3 \times 9)$

C $(7 \times 3) + (3 \times 9)$

D 7×27

E $21 + 9$

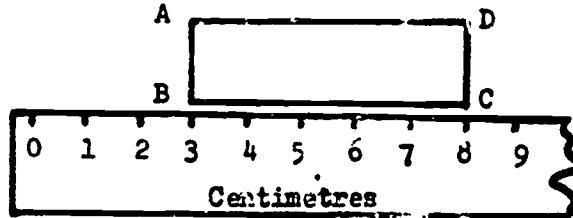
174.



PQRS is a rectangle. Its image after a transformation is the rectangle P'Q'R'S', as shown above. The transformation used could have been

- A a rotation about the origin
- B a reflection in the y -axis
- C a translation parallel to the x -axis
- D a reflection in the x -axis
- E a translation parallel to the y -axis.

175.



According to the scale shown, the length of side BC of the rectangle ABCD (to the NEAREST CENTIMETRE) is

- A 5 centimetres
- B 6 centimetres
- C 7 centimetres
- D 8 centimetres
- E 9 centimetres

176. $-5(6 - 4)$ is equal to

A 50

B 26

C 10

D -10

E -26

177. \$150 is divided in the ratio of 2 to 3. The smaller of the two amounts is

A \$30

B \$50

C \$60

D \$90

E \$120

178.

847.36

In the number in the box the digit 6 represents

A $6 \times \frac{1}{100}$

F $6 \times \frac{1}{10}$

C 6×1

D 6×10

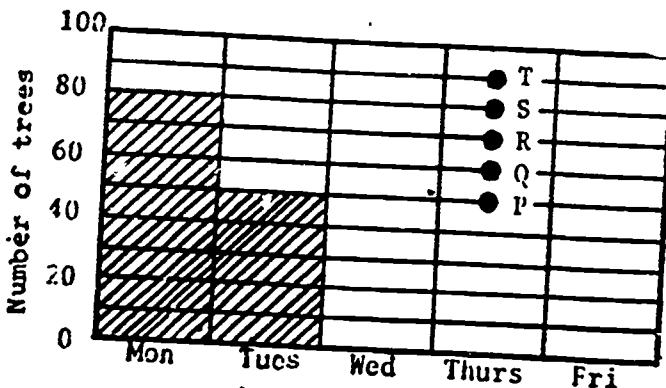
E 6×100

97

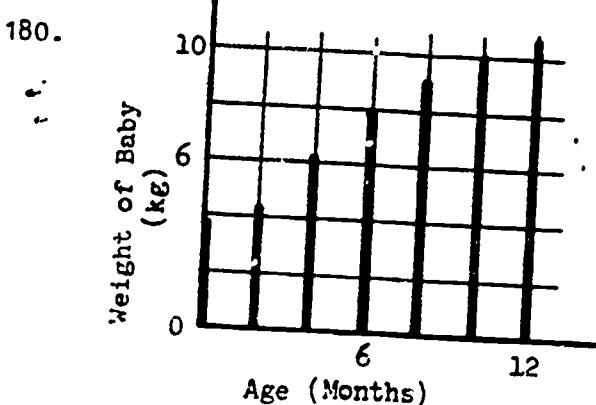
179. Here is a table that shows the number of trees planted along a highway in a week.

Days of the Week	Mon	Tues	Wed	Thurs	Fri
No of Trees Planted	80	50	60	90	75

On the diagram below the graph for the first two days' plantings has been drawn. If the graph were completed, which point would indicate the top of the bar on Thursday?



A P
 B Q
 C R
 D S
 E T



The weight gain from 6 to 10 months was

A 1 kg
 B 2 kg
 C 4 kg
 D 6 kg
 E 8 kg

APPENDIX II
DATA MATRIX

TEST

ITEM	BFLBFRCBCCONENGFINRAHONHUNISRJAPLUXNTHNWZSCOSWETHAUSA	OTL
1	71 77 50 43 34 63 84 44 57 41 47 74 65 32 38 64 31 40	93 0 82 91 93 94 100 0 96 60 97 94 92 83 0 94 85 76
3	45 44 18 12 22 19 42 24 46 27 58 22 31 14 25 14 16 21	88 0 77 70 69 47 91 0 98 77 97 55 79 55 0 48 92 56
4	35 46 39 31 38 27 27 25 31 34 33 29 44 38 41 27 19 30	97 0 86 96 92 94 94 0 98 62 98 93 88 88 0 81 84 84
5	59 45 47 57 44 52 66 50 65 43 68 44 58 33 44 48 41 42	100 0 80 97 88 78 98 0 99 71 88 80 81 78 0 80 89 91
6	77 65 73 66 68 65 78 69 71 55 75 50 64 66 57 65 71 53	83 0 67 82 83 64 98 0 91 40 89 40 74 79 0 59 65 70
7	57 72 74 67 67 62 75 52 77 54 82 15 82 70 70 13 39 61	85 0 28 29 65 22 65 0 91 41 91 54 80 44 0 19 35 26
8	91 85 94 93 88 90 96 80 95 73 87 71 89 94 88 81 86 87	79 0 88 93 81 88 98 0 93 57 93 67 84 80 0 81 70 83
9	42 31 44 50 43 32 62 43 43 27 70 21 44 37 37 36 31 47	85 0 81 91 92 80 99 0 99 63 97 64 78 73 0 87 85 84
10	23 20 13 15 16 15 15 17 14 25 35 19 20 13 15 12 5 10	79 0 41 56 55 48 98 0 98 41 73 77 59 37 0 25 79 56
11	34 37 40 37 26 16 36 17 22 24 30 14 60 40 29 25 21 29	89 0 70 64 54 67 83 0 91 58 87 39 87 77 0 65 83 55
12	55 56 52 57 42 38 55 50 49 32 60 30 54 44 44 30 63 40	60 0 77 96 67 42 87 0 99 56 96 63 58 44 0 37 95 89
13	61 71 72 72 55 60 89 70 69 57 81 58 73 61 63 48 58 68	85 0 89 91 83 78 100 0 96 83 93 62 85 87 0 44 96 87
15	28 24 27 37 32 33 38 31 41 23 37 27 34 18 33 35 22 30	100 0 89 90 71 92 99 0 96 65 88 70 81 49 0 83 87 90
16	67 71 68 61 44 73 77 52 68 76 85 59 73 46 59 22 63 55	92 0 96 91 91 97 100 0 99 96 98 62 92 99 0 52 99 93
17	72 68 57 76 55 69 59 67 76 54 62 52 68 59 55 77 65 53	89 0 76 94 98 63 94 0 98 66 82 84 93 100 0 84 95 83
18	43 49 56 52 36 48 56 43 43 60 50 29 41 50 34 28 36 48	91 0 90 90 80 78 94 0 97 80 98 58 78 82 0 64 98 77
19	39 37 32 42 33 38 29 40 71 30 75 15 42 42 34 37 42 31	17 0 14 27 32 30 59 0 94 27 92 32 22 17 0 14 28 23
20	78 77 79 83 69 78 92 82 89 73 89 73 85 63 75 73 72 73	75 0 93 100 100 96 100 0 96 62 98 94 93 100 0 96 95 97
21	29 32 33 41 46 36 29 42 43 29 50 14 47 42 49 37 35 30	14 0 0 59 40 28 16 0 97 27 21 15 63 62 0 30 68 39
22	59 69 65 72 74 67 54 72 79 52 87 48 74 75 74 70 72 53	85 0 81 85 95 44 19 0 97 61 93 64 88 100 0 79 86 81
23	69 76 57 60 59 58 75 70 70 52 87 77 78 55 62 56 59 44	84 0 73 91 92 64 100 0 96 68 98 95 83 85 0 73 95 79
25	64 47 57 48 48 44 52 42 58 36 61 38 73 54 56 51 22 43	92 0 92 94 91 87 100 0 100 93 98 73 94 96 0 84 79 90
26	55 58 59 64 49 58 74 55 75 60 62 67 65 40 52 56 49 60	91 0 94 100 100 96 100 0 98 75 97 94 94 99 0 95 97 100
27	74 64 64 57 62 68 84 61 78 61 73 46 68 57 67 51 67 54	86 0 70 42 57 56 94 0 99 78 96 30 60 38 0 29 93 44
28	67 61 65 57 49 53 43 68 49 50 75 44 55 50 57 46 50 57	95 0 88 93 98 89 100 0 99 62 99 89 89 89 0 86 93 88
29	87 83 93 89 81 88 94 60 94 74 90 84 86 89 79 83 88 71	86 0 90 99 87 87 100 0 96 63 96 88 93 100 0 87 89 87
30	51 43 34 37 42 45 38 40 66 34 62 30 50 37 44 41 28 38	95 0 57 38 45 35 79 0 97 47 78 30 31 32 0 17 41 43
31	69 73 68 62 41 45 72 69 67 67 88 61 65 38 55 27 47 57	97 0 94 100 100 96 100 0 100 80 98 91 96 100 0 74 98 100
32	46 46 52 45 27 42 54 61 61 52 68 36 47 24 32 30 35 41	87 0 93 100 93 96 100 0 97 68 98 89 91 96 0 87 97 100
33	49 16 61 61 42 20 15 51 55 26 31 15 47 51 43 24 60 47	63 0 79 93 77 61 98 0 100 66 97 58 66 39 0 40 97 91
36	45 57 61 60 48 42 36 48 59 51 46 37 58 48 56 47 29 50	88 0 84 93 89 58 93 0 99 84 98 77 85 61 0 76 86 90
37	48 55 50 31 36 47 26 38 52 40 57 19 52 34 42 37 33 34	88 0 88 82 76 93 94 0 98 82 96 58 92 91 0 81 96 83
38	59 64 52 50 45 57 41 66 68 61 62 54 76 43 56 64 41 48	93 0 89 97 87 79 97 0 98 90 98 85 89 72 0 92 94 98
39	30 25 59 46 49 69 59 39 66 35 80 27 77 62 70 49 42 50	12 0 73 61 88 82 62 0 100 80 94 33 95 96 0 60 59 64
40	50 46 50 46 57 41 67 60 52 40 69 12 61 48 55 43 42 44	83 0 37 48 44 23 76 0 99 13 4 15 16 29 0 8 53 48
41	86 76 77 76 67 59 78 51 70 64 70 64 71 70 65 56 40 76	97 0 91 99 89 92 100 0 97 55 97 94 93 91 0 87 87 89
42	75 65 74 74 79 81 61 57 87 51 68 27 88 88 78 80 48 70	27 0 27 40 68 43 90 0 100 22 58 8 91 99 0 10 25 27
44	34 31 23 21 30 22 33 30 21 15 35 15 33 20 28 25 12 18	85 0 79 90 73 77 90 0 94 66 99 80 82 67 0 78 85 79

TEST

ITEM	BFLBFRCBCCONENGFINRAHONHUNISRJAPLUXNTHNWZSCOSWETHAUSA	OTL
46	23 30 30 29 37 31 16 34 49 32 49 45 30 26 40 41 23 18	11 0 71 68 51 30 17 0100 39 30 31 62 60 0 63 80 55
47	91 90 84 81 84 90 95 76 95 85 91 83 91 78 82 85 79 66	98 0 87 99 98 97100 0 96 62 98 93 94 94 0 95 97 83
48	46 31 15 18 24 17 32 19 0 6 15 5 12 20 15 15 22 21	74 0 6 1 10 8 8 0 0 4 0 6 4 5 0 2 15 3
49	43 45 42 46 42 49 49 48 50 42 62 48 62 29 44 44 39 35	88 0 66 90 79 59 99 0 91 58 96 93 73 58 0 43 96 80
50	33 32 32 32 31 32 24 37 47 32 57 15 36 36 28 28 37 32	15 0 67 71 69 52 22 0 94 41 39 49 81 90 0 50 75 59
51	80 72 82 81 83 84 84 78 96 78 85 73 82 83 76 79 49 82	91 0 86 98 86 85 85 0 97 63 92 82 62 79 0 70 82 97
52	52 35 45 41 46 62 48 32 60 47 34 29 45 43 49 63 42 51	44 0 47 59 65 54 73 0 91 27 44 22 21 74 0 37 41 72
54	23 25 20 20 23 30 32 24 42 18 34 13 35 27 22 24 16 24	17 0 12 13 17 29 80 0 76 10 66 22 32 19 0 14 31 7
55	62 60 74 63 58 69 64 59 56 45 87 48 68 66 57 75 42 60	40 0 18 19 30 22 40 0 53 12 68 14 42 13 0 13 46 19
56	18 26 20 18 22 20 22 25 26 25 51 7 22 17 23 18 27 22	27 0 68 69 33 28 97 0 98 49 95 52 33 18 0 6 63 63
57	45 41 38 39 39 38 47 45 39 50 41 37 50 40 35 32 29 36	49 0 26 30 43 43 35 0 42 21 20 18 24 29 0 22 26 21
58	64 72 65 63 76 68 81 64 67 64 76 49 71 62 68 72 58 60	76 0 82 77 82 85 93 0 97 68 97 75 79 65 0 93 86 81
59	66 66 65 51 70 68 70 64 66 42 84 50 74 65 67 50 55 57	20 0 46 53 60 35 65 0 98 51 91 50 40 34 0 51 65 63
60	31 34 36 35 30 35 19 48 41 31 65 41 41 35 35 34 26 35	94 0 67 85 68 39 96 0 98 67 96 89 64 65 0 43 95 82
61	41 36 52 61 27 46 55 37 65 56 59 51 52 25 30 39 24 61	87 0 0 100 91 97100 0 97 62 96 94 89 80 0 93 96100
62	79 84 88 83 91 88 87 77 94 86 89 58 90 87 89 88 77 81	61 0 57 77 88 70 69 0 97 55 98 33 41 77 0 52 73 81
64	61 63 78 65 58 73 75 54 48 55 58 53 67 66 56 48 50 65	79 0 93 69 67 80100 0 48,54 56 67 76 50 0 29 66 75
66	47 52 50 44 31 54 57 31 50 41 63 32 55 33 44 14 33 38	88 0 91 90 81 91100 0 99 97 98 57 84 93 0 42 98 82
67	53 62 53 60 58 66 51 66 71 55 84 42 77 65 64 60 55 57	21 0 68 76 83 52 40 0 98 56 58 59 84 96 0 56 84 61
68	48 60 59 62 47 56 56 54 30 51 62 41 66 43 54 42 39 55	84 0 87 94 77 78 93 0100 86 92 52 77 73 0 44 87 72
70	31 42 50 46 36 19 28 45 38 34 7 36 59 37 42 19 12 43	7 0 54 68 62 4 8 0 37 39 1 58 65 22 0 4 37 58
71	53 66 46 47 59 51 55 63 68 52 78 68 71 52 67 46 51 46	77 0 70 86 85 53 92 0 97 65 92 85 75 79 0 61 94 70
72	66 70 53 56 59 62 62 59 63 44 76 54 73 59 59 61 48 53	76 0 74 81 69 67 64 0 99 78 94 71 74 45 0 61 87 62
73	28 28 34 33 28 28 49 27 20 20 40 13 30 23 31 30 19 32	20 0 80 56 46 37 64 0 18 44 15 19 36 18 0 28 73 71
74	13 13 23 19 26 39 9 24 37 26 34 8 50 25 41 21 18 23	2 0 53 36 48 61 43 0 94 62 82 24 80 58 0 44 36 37
75	57 56 49 45 50 71 35 46 64 71 68 23 88 50 64 40 36 52	33 0 90 80 92 96 92 0 96 95 98 62 97 93 0 91 92 84
78	51 51 55 50 51 55 53 43 46 37 52 28 55 44 59 56 23 54	51 0 38 56 50 46 72 0 53 26 37 13 16 65 0 23 48 67
79	19 21 18 19 25 20 11 34 24 19 41 10 16 22 18 19 26 18	15 0 20 39 60 24 40 0 98 21 95 12 12 45 0 8 51 30
80	28 26 27 21 28 27 37 26 46 27 49 20 39 27 34 14 15 24	33 0 61 38 44 44 76 0 97 63 58 19 38 45 0 10 26 36
81	61 47 57 63 47 58 63 50 56 45 66 33 60 63 56 45 56 44	25 0 42 54 60 35 43 0 98 29 69 17 82 97 0 19 60 34
82	55 53 44 39 35 56 58 42 58 38 45 44 62 36 41 50 30 38	96 0 81 92 92 90 86 0100 77 96 90 90 85 0 88 88 86
83	27 32 29 17 35 40 18 42 48 30 51 22 21 36 35 31 39 16	14 0 60 71 67 38 26 0 94 41 52 32 72 76 0 35 66 42
84	78 76 76 77 78 76 89 73 86 78 81 74 84 73 79 75 71 72	66 0 78 97 86 77 87 0 97 74 94 85 44 73 0 52 59 94
85	48 57 34 62 25 42 66 39 48 40 34 21 34 17 32 20 34 60	98 0 90 99 77 95100 0 92 72 36 62 84 71 0 36 92100
86	19 15 13 16 18 17 5 16 11 6 12 3 12 16 15 17 14 16	26 0 1 6 13 5 16 0 0 2 3 6 7 11 0 4 10 2
87	55 61 43 38 40 39 58 49 59 51 77 37 54 36 45 32 40 39	59 0 87 82 81 47 63 0100 84 98 64 90 55 0 61 92 72
88	23 29 27 32 37 33 16 38 40 32 59 9 44 32 45 34 28 19	20 0 47 62 70 36 31 0 98 39 60 43 80 72 0 48 70 44
89	55 40 14 43 31 56 16 34 55 31 62 8 48 31 32 43 20 37	23 0 11 30 31 31 16 0 97 19 22 4 65 33 0 4 39 16
90	62 61 55 56 49 50 69 51 67 58 47 43 56 53 49 37 41 53	93 0 86 82 74 88 99 0 91 68 56 67 84 85 0 74 88 84

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ITEM	BFLBFRCBCCONENGFINRAHONHUNISRJAPLUXNTHNWZSCOSWETHAUSA	BFLBFRCECCONENGFINRAHONHUNISRJAPLUXNTHNWZSCOSWETHAUSA
91	68 72 66 64 36 61 81 66 61 63 78,69 56 48 34 13 35 56	95 0 91 99 93 94100 0 96 86 98 90 85 91 0 25 94 92
92	45 59 42 41 57 52 46 47 66 39 75 51 56 42 49 51 50 34	66 0 83 90 92 66 78 0 98 55 95 80 90 99 0 82 84 76
93	56 47 38 35 43 40 50 49 44 34 74 43 59 40 44 43 31 23	83 0 69 65 75 51100 0 94 50 95 81 74 68 0 53 76 58
94	56 35 56 48 46 43 57 43 45 44 63 17 54 47 45 45 34 49	19 0 36 44 58 41 48 0 86 54 81 28 28 32 0 52 56 68
95	72 63 77 60 51 19 72 83 44 71 75 63 58 69 65 47 59 64	95 0 91100 94 46100 0 89 71 98 91 87100 0 52 85 97
96	79 83 54 63 58 69 84 76 70 63 82 72 69 53 59 47 51 55	98 0 0 84 87 62 90 0 99 56 70 86 93 89 0 70 95 82
97	28 20 30 22 14 17 37 21 16 19 10 12 17 17 13 19 28 20	55 0 82 59 43 47 83 0 30 52 6 18 31 33 0 20 55 68
98	65 73 66 57 69 66 63 74 70 61 64 64 67 61 66 62 67 56	79 0 81 63 81 59 96 0 97 60 95 81 72 51 0 69 89 89
99	66 72 32 17 45 61 78 46 66 63 73 46 73 29 53 42 18 29	.76 0 67 64 93 88100 0 98 88 79 63 89 82 0 50 69 49
100	15 19 42 20 36 15 10 20 38 29 22 11 53 23 44 17 18 26	3 0 69 44 60 4 5 0 37 38 1 27 57 33 0 3 36 55
103	30 32 28 34 31 21 23 35 48 32 48 27 47 31 34 26 18 25	87 0 76 80 87 50 98 0 98 72 97 93 73 77 0 63 85 69
104	72 75 56 50 44 72 80 59 76 77 70 54 75 37 51 56 48 49	87 0 82 91 76 82 98 0 99 90 98 59 83 77 0 59 97 84
105	51 53 48 42 33 42 70 43 44 39 71 55 41 29 37 24 32 40	95 0 93 92 87 95100 0 96 94 98 70 85 99 0 51 97 93
106	57 43 57 61 54 72 47 44 65 33 46 9 75 46 61 63 35 51	51 0 80 91 77 94 76 0 98 49 70 36 82 58 0 88 78 72
108	38 48 57 49 50 21 40 46 41 43 19 43 68 45 53 22 28 48	11 0 71 80 68 8 17 0 33 45 0 66 67 37 0 2 39 61
109	64 80 69 72 66 71 77 75 87 81 83 79 73 61 67 64 64 66	98 0 94100100 97100 0 99 88 98 89 93 97 0 75 97 97
110	68 72 59 52 55 60 52 6,5 56 60 72 67 77 50 65 59 39 47	96 0 91 99 96 93100 0 97 77 98 93 89 89 0 81 96 92
111	31 37 33 21 29 30 32 26 56 29 24 26 53 23 36 34 21 25	8 0 73 51 60 3 5 0 40 41 2 30 60 28 0 6 40 56
112	95 91 65 61 35 94 98 54 97 89 93 92 92 45 36 93 86 37	58 0 68 76 53 70 49 0 77 47 90 52 71 79 0 64 54 62
113	87 85 86 83 63 60 80 75 74 77 82 78 86 64 71 46 73 78	95 0 94100100 95100 0100 80 98 91 96100 0 83 97100
115	78 61 63 54 43 50 70 66 57 60 83 59 71 34 52 36 54 57	98 0 96100 98 95100 0 99 71 98 93 93 91 0 79 97 99
116	52 46 42 40 40 47 38 47 28 40 49 30 55 52 45 54 24 45	35 0 22 29 40 4 10 0 26 12 6 6 16 49 0 6 15 39
117	70 48 81 73 74 67 62 57 63 61 74 30 78 80 67 64 66 74	23 0 20 35 39 21 9 0 31 18 44 15 20 41 0 23 34 35
118	80 76 78 74 78 67 67 71 79 74 89 47 81 76 73 67 75 75	81 0 81 88 91 79 89 0 87 66 73 68 82 84 0 81 74 87
119	14 13 36 25 27 18 17 25 60 20 26 10 27 28 24 26 46 28	8 0 52 44 39 20 3 0 99 15 3 9 16 37 0 12 64 47
120	62 52 49 48 59 56 58 59 63 45 80 37 69 50 54 55 50 59	18 0 43 46 63 44 47 0 88 54 84 27 29 35 0 55 55 69
121	23 29 34 29 34 32 9 22 47 11 28 10 39 32 35 22 32 34	5 0 70 58 73 75 55 0 98 68 88 30 90 87 0 54 40 58
122	45 64 53 44 49 50 29 37 52 54 68 37 42 45 43 58 54 49	60 0 78 70 51 49 62 0 98 82 96 28 50 44 0 33 70 59
123	56 45 61 50 36 42 46 34 28 39 41 23 45 43 43 34 50 57	68 0 80 79 60 53 76 0 96 80 96 27 57 46 0 26 93 61
125	54 42 22 20 16 35 38 16 39 26 33 29 41 23 16 19 22 15	100 0 50 80 75 56 91 J 99 51 98 94 83 58 0 41 93 71
126	45 49 41 35 32 45 48 41 43 47 46 26 51 31 36 37 30 36	73 0 88 79 74 85 94 0 99 84 90 57 75 76 0 33 93 65
127	54 64 50 56 55 45 71 58 61 53 67 69 69 45 55 48 67 44	96 0 62 80 81 52100 0 96 54 97 93 76 57 0 32 86 66
128	69 61 68 65 53 54 52 58 56 45 51 30 59 50 56 48 54 55	87 0 82 92 70 42100 0 98 52 87 49 54 41 0 28 96 90
129	60 51 41 38 43 54 54 50 50 42 72 45 52 41 43 50 38 41	66 0 53 58 64 53 93 0 89 39 96 68 59 54 0 52 53 46
130	18 17 26 18 16 11 9 30 36 13 52 11 20 17 22 12 16 17	88 0 53 79 73 54100 0 98 47 97 86 73 55 0 33 87 62
131	59 54 45 52 34 49 62 49 45 36 63 45 52 28 38 31 40 43	76 0 0 96 58 75100 0 76 68 77 68 51 38 0 28 96 86
132	59 61 60 50 59 73 61 63 72 62 66 57 75 48 60 66 53 52	52 0 69 79 56 48 93 0 98 66 93 72 58 48 0 71 86 73
133	70 58 38 38 41 39 47 44 60 40 52 32 39 44 35 41 54 35	65 0 76 73 69 41 82 0 99 61 20 66 92 78 0 55 92 74
134	45 53 43 31 29 42 63 44 54 44 59 33 37 31 32 24 37 35	83 0 80 56 51 71100 0 81 69 56 46 73 55 0 24 92 65

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ITEM	BFLBFRCBCCONENGFINRAHONHUNISRJAPLUXNTHNWZSCOSWETHAUSA	BFLBFRCBCCONENGFINRAHONHUNISRJAPLUXNTHNWZSCOSWETHAUSA
136	67 62 75 65 69 62 70 73 79 62 84 36 73 67 58 58 67 73	45 0 77 65 68 53 37 0 96 66 90 45 43 66 0 45 63 75
137	44 43 51 46 42 55 50 44 54 44 67 31 67 39 42 46 19 47	45 0 50 71 71 53 47 0 98 44 80 14 19 67 0 39 48 70
138	76 86 83 83 83 84 91 81 93 91 89 89 86 75 78 78 80 83	100 0 92100100 98100 0 96 54 98 93 92 96 0 96 96100
139	45 44 53 44 40 47 47 38 71 44 74 31 59 39 40 63 60 43	15 0 70 58 46 31 40 0 100 40 35 13 79 69 0 56 87 55
140	51 34 45 45 36 41 37 55 55 49 71 29 58 36 40 46 22 44	80 0 68 82 91 77 70 0 100 72 96 72 60 74 0 51 20 87
141	37 50 33 29 40 34 29 63 59 38 78 45 50 35 43 37 30 28	84 0 61 67 73 30 98 0 100 71 98 93 68 62 0 51 94 65
142	47 66 69 65 61 63 75 54 78 47 79 33 68 70 60 54 61 57	3 0 17 30 40 31 5 0 93 13 44 3 64 76 0 4 42 18
143	27 45 32 29 31 11 28 35 19 42 9 32 43 30 30 13 8 33	8 0 53 77 62 10 8 0 36 46 0 54 64 35 0 2 49 60
144	39 50 33 34 43 44 31 59 49 36 64 26 46 32 40 44 42 31	19 0 72 76 83 50 40 0 97 56 65 58 80 94 0 56 82 52
145	46 55 27 26 24 23 54 58 47 46 68 24 29 29 27 15 42 28	85 0 43 48 48 65100 0 98 78 87 40 59 58 0 17 94 39
147	66 74 68 64 61 59 75 74 69 63 88 80 86 51 63 52 82 50	91 0 68 90 92 59100 0 97 58 98 85 84 68 0 37 93 81
148	34 43 25 23 29 29 34 43 47 30 38 33 44 22 27 31 36 21	51 0 76 84 70 43 97 0 94 77 98 77 59 38 0 71 94 81
149	32 40 42 30 48 61 41 21 48 29 67 22 67 53 58 39 14 35	7 0 64 47 64 55 49 0 86 69 58 28 80 72 0 46 41 47
150	46 36 63 55 33 53 58 23 45 36 54 32 57 56 47 29 39 49	85 0 93 91 57 89 98 0 97 73 94 63 91 94 0 86 93 81
151	42 40 23 14 12 19 54 30 29 24 48 28 23 19 14 9 21 21	93 0 90 94 73 93100 0 96 89 97 70 87 84 0 53 93 88
152	58 44 56 50 40 54 59 56 53 30 50 32 47 53 47 49 55 42	68 0 60 70 76 62 84 0 83 39 81 22 62 73 0 46 63 47
153	48 46 63 66 59 51 58 49 50 33 78 23 51 67 64 45 53 65	14 0 39 40 64 45 36 0 61 24 43 6 16 76 0 30 47 64
155	67 67 69 62 66 62 74 62 53 68 67 64 78 60 66 70 50 61	85 0 84 92 96 91 91 0 98 86 97 85 91 87 0 90 80 87
156	56 49 58 55 60 60 55 54 44 42 30 43 67 60 64 62 40 53	35 0 53 70 87 52 47 0 96 45 73 15 24 73 0 52 61 79
157	51 52 50 45 51 32 60 54 70 54 61 48 53 50 50 45 35 40	33 0 27 41 46 31 24 0 66 18 44 17 36 44 0 25 36 30
158	73 77 86 86 79 75 91 71 86 65 82 75 82 78 80 70 79 84	100 0 93100 98 95100 0 97 57 97 95 96100 0 92 97 99
159	49 46 58 51 46 46 42 52 47 43 59 37 61 45 48 44 42 46	89 0 90 96 81 69 94 0 98 79 98 80 70 61 0 85 96 91
160	60 75 52 48 43 43 49 53 58 48 69 52 60 44 47 35 56 41	45 0 56 69 50 20 30 0 100 44 51 61 26 28 0 16 54 61
161	61 59 76 71 81 82 55 70 83 81 91 59 75 74 73 75 79 69	15 0 76 74 46 47 47 0 98 60 47 26 76 73 0 78 88 73
162	54 43 22 22 28 28 37 23 19 26 37 29 34 34 26 24 22 25	93 0 57 40 48 74 97 0 43 61 54 70 59 84 0 18 12 42
163	28 23 13 22 28 26 17 26 13 8 25 8 27 24 20 21 25 25	11 0 0 1 13 4 52 0 1 2 1 4 7 11 0 1 8 3
164	53 57 64 68 52 72 39 66 74 62 79 37 68 66 53 68 64 52	20 0 60 52 75 34 44 0 96 51 82 44 62 89 0 33 75 44
165	63 56 77 66 58 60 79 56 65 50 73 51 66 71 59 40 67 68	81 0 83 68 55 48 97 0 96 75 85 35 74 65 0 24 86 69
166	81 82 86 79 86 84 83 79 89 85 88 66 87 83 80 85 78 76	29 0 0 70 93 53 40 0 94 55 93 21 35 79 0 63 69 82
167	38 60 35 48 43 42 28 43 52 40 65 33 63 61 58 42 38 40	7 0 20 33 49 31 92 0 92 13 53 12 68 85 0 10 19 22
168	62 76 49 46 47 64 67 69 79 69 74 77 80 44 49 62 57 37	100 0 82 27 95 94100 0 96 65 98 97 93 89 0 93 94 76
169	34 40 22 26 37 33 15 33 53 36 45 17 43 37 36 13 33 21	26 0 59 74 83 31 28 0 99 51 61 33 72 79 0 43 79 57
171	43 39 37 37 31 34 27 41 36 31 45 17 40 35 33 35 44 44	69 0 78 62 38 25 68 0 79 72 20 13 62 50 0 43 97 54
172	49 36 35 24 37 38 39 40 48 47 61 28 48 31 37 33 26 29	67 0 56 28 49 52 61 0 100 74 96 17 50 28 0 24 80 37
173	65 67 59 48 41 49 80 60 64 47 73 56 59 48 50 23 45 47	95 0 90 96 83 92100 0 99 96 97 77 88 90 0 46 96 85
174	28 36 27 32 33 40 18 31 50 17 58 13 53 47 45 24 29 24	6 0 10 27 38 30 23 0 96 7 30 4 67 71 0 5 17 9
175	66 85 79 65 70 84 87 63 88 62 89 66 86 65 69 78 67 54	85 0 87 94 98 92 97 0 98 52 96 82 92 93 0 92 84 90
176	65 64 74 63 59 63 75 62 67 62 78 61 66 61 55 44 60 63	91 0 94 87 83 95 99 0 98 89 98 61 85 91 0 44 97 87
178	75 70 70 64 49 56 58 52 58 45 72 42 70 51 54 47 37 57	85 0 92 98100 90 93 0 97 74 91 90 93 97 0 90 89 94
179	53 64 71 68 85 78 66 73 88 79 79 46 89 71 83 80 57 67	28 0 51 67 96 57 45 0 99 62 91 36 38 73 0 61 55 77
	38 54 57 41 49 55 27 51 47 49 78 32 56 44 52 61 39 43	33 0 52 63 85 62 45 0 97 56 95 23 32 74 0 60 64 82 107